

Tweaking the Transistor

To what extent do temperature and voltage influence the peak switching frequency in mosfet transistors?

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Abstract

While under the ever growing demand of computational power, decreasing the size of the transistor is becoming increasingly difficult. To keep raising the power of computers it is, now more than ever, important to tweak the transistors in our chips. This paper studies the relations between the temperature and operating voltages of a power mosfet and the maximum switching frequency it can achieve.

From the regarded theory the following relation was derived and predicted:

$$f_{max} = a * \frac{(V_{gs} - V_{th})^2}{\left(\frac{T_j}{300}\right)^{2.3} * \ln(C * V_{ds} + \Delta V)}$$

After conducting two experiments that form a way of estimating the temperature inside the transistor by measuring difference in ambient and case temperature, the transistor is placed in an environment where all variables can be altered and measured. In total 145 datapoints were collected pertaining all variables and the maximum achieved frequency. Using regression analysis the following formula was derived that fits the data to an R^2 of 0.02928:

$$f_{max} = \frac{4.565}{\left(\frac{\left(\frac{T_j}{300}\right)^{-0.476}}{(V_{gs} - 3.1)^2} - 0.0326\right) * \ln(35.666 * V_{ds} - 4.866)}$$

The analysis does seem to confirm the hypothesis about the relation between source-drain voltage. The relation between the gate voltage and maximum frequency shows too. Although the accumulation of measurement inaccuracy prevents making solid statements about the temperature correlation, the analysis seems to reject the hypothesized relation, showing instead an increase in maximum frequency when the temperature and resistance increase.

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We thank you for taking the effort to read this paper and wish you learn something new or learn us something new. Feel free to contact us to give or receive any further information, explanation or questions.

With kind regards,
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Table of Abbreviations

Abbr.	Full name	Description
V_{gs}	Gate-source voltage	The voltage difference between the gate and the source of the transistor.
V_{ds}	Drain-source voltage	The voltage difference between the drain and the source of the transistor.
V_{th}	Threshold voltage	The minimum value for V_{gs} to turn on the mosfet, allowing current to flow between the drain and the source.
V_{ps}	Power supply voltage	The voltage applied over the source and drain by the power supply (differs from V_{ds} due to power drop over resistors)
I_{ds}	Drain-source current	The current running from the drain to the source of the mosfet.
T_j	Junction temperature	The temperature of the junction inside the mosfet.
T_c	Case temperature	The temperature of the case of the mosfet.
T_a	Ambient temperature	The temperature of the air around the mosfet.
f_{max}	Maximum frequency	The highest frequency the transistor can switch at while still reaching (near) 0 V voltage drop in between switches.

Introduction

In 1965 Gordon E. Moore wrote the article “Cramming More Components onto Integrated Circuits”, which led to the now famous Moore’s law, predicting a doubling of the number of components per integrated circuit every year. This rule of thumb, eventually revised to predict the doubling of transistors in an IC every two years, is an example of the massive amount of growth the semiconductor industry has undergone since then. Technology now plays a central role in our lives and at the heart of it all we find the transistor.

With headlines such as “Moore’s Law Is Dead. Now What?” - *MIT Technology Review* and Intel slowing down their iteration cycle being just examples, it can be said that, in 2018, Moore’s law no longer holds true. This does however not mean the search for ever-increasing performance is over. If cramming more transistors in an IC is no longer an option, we will have to run those transistors at a higher frequency. That is, of course, easier said than done. Many factors - the size of the transistor, the operating voltage and temperature, just to name a few - play a role.

In order to improve the switching speed we need to study all the factors that can limit the switching frequency. It is for this reason that a few of them have been chosen to be examined and hopefully understood and charted.

The transistors we will be looking at are mosfets. Today they are the most common type of transistor, used in a wide range of applications like signal amplification, power supply and computing. Their maximum switching frequency is heavily influenced by temperature, voltage and impedance. These are some of the most important environmental factors, that’s why we have chosen them to study first. So in this paper you will find *to what extent do voltage, temperature and impedance influence peak switching frequency in mosfet transistors.*

In the world of electrical engineering, almost all factors influence each other. Because of this, we will first take a look at the individual roles. Later on, we will look at the bigger picture and try to explain the way equilibrium between all factors is reached, ending with what we hope will be an accurate model of the properties of a transistor.

The importance of a proper balance between the factors was showcased by Apple’s recent release of its 2018 model Macbook Pro. The newer model, with its bigger and theoretically faster chip, was outperformed by its older brother from 2016. This happened because Apple had improperly managed the new chip’s thermal performance, making it require a significant drop in voltage, accompanied by a drop in frequency to maintain stability and thus leading to performance loss.

We hope you’ll be able to explain exactly why this happened by the end of this paper.

Hypothesis

1. Combining formulas

As the range for V_{gs} is rather small and only slightly above V_{th} , the quadratic relation between I_{ds} and V_{gs} is expected to be seen, although this would eventually become linear.

Applying equation (3.4) to the relation described in equation (3.1), the relation in equation (1.1) can be derived, where n is a multiplier required to solve the equation.

$$R_{25C} = n * \frac{1}{k_{25C}(V_{gs} - V_{th})^2} \quad (1.1)$$

Equation (3.1) was chosen over (3.2) as the range for V_{gs} is rather small and close to V_{th} , which means the quadratic relation between I_{ds} and V_{gs} is expected to be seen.

This can be combined with equation (3.3) to yield equation (1.2).

$$R = n * \frac{\left(\frac{T_j}{300}\right)^{2.3}}{k_{25C}(V_{gs} - V_{th})^2} \quad (1.2)$$

Equation (1.2) can in turn be used to substitute R in equation (3.13), yielding equation (1.3).

$$f_{max} \propto \frac{1}{n * \frac{\left(\frac{T_j}{300}\right)^{2.3}}{k_{25C}(V_{gs} - V_{th})^2} * C * \ln(C * V_{ds})} \quad (1.3)$$

Adding an offset ΔV to the logarithm in equation (1.3), as $1/\ln(1)$ does not exist, and combining constants wherever possible to finally yield equation (1.4), which will function as the hypothesis.

$$f_{max} = a * \frac{(V_{gs} - V_{th})^2}{\left(\frac{T_j}{300}\right)^{2.3} * \ln(C * V_{ds} + \Delta V)} \quad (1.4)$$

Theory

Before continuing to the core of the research, the groundwork will have to be laid out. In this section, a close look will be taken at the theoretical aspects of transistors, starting with the uses and functioning, advancing to the relevant variables and taking a look at the thermodynamics at play to eventually form a hypothesis.

2. Transistors

Uses

Transistors are a common electrical component found in a wide range of electronic appliances, ranging from audio equipment to power supplies to the chips in mobile phones.

At its lowest level, a transistor simply blocks any current from passing from the drain and the source, or allows the current to flow freely. Whether or not current is allowed to flow from one end to the other, depends on the voltage applied to the gate (the gate-source voltage, or V_{gs}). Once a certain threshold voltage is passed (V_{th}), the transistor switches from one state to the other (on to off, or off to on, depending on the type of transistor). If the voltage is lowered below the threshold voltage, the transistor once again reverts to its original state.

Even when V_{gs} is above the threshold voltage, further increases in the voltage change the conductivity of the transistor, either lowering or raising it further.

This property is commonly used to amplify signals, as the voltage drop over the drain and source (V_{ds}) will scale with the resistance, which in turn scales with the input signal on V_{gs} , allowing you to amplify weak signals, like those received from radio waves, to stronger signals usable for speakers.

Transistors also find a use in power supplies, where their ability to quickly change resistance at a moment's notice allows them to precisely limit the current. Combined with inductors and capacitors to smooth out the output voltage, a higher voltage can be lowered with relatively little power loss.

Last but certainly not least, transistors are used in logic gates to perform binary operations. A single transistor can be viewed like an AND-gate, only outputting a positive voltage if a voltage is applied to the gate *and* drain, of which large amounts can be combined to perform more complex operations

A common type of transistor is a MOSFET (metal-oxide-semiconductor field-effect-transistor).

Semiconductors

Mosfets derive the ability to change their conductivity from the material they are made of. Whereas most materials are either conductors that are able to conduct current, or insulators that are unable to conduct current, some materials share characteristics of both groups and are aptly named semiconductors. The resistance of semiconductors can be orders of magnitude higher than conductors, yet still orders of magnitude lower than resistors. [1]

This resistance can be customized by adding trace amounts of impurities that

function as a charge carrier. These carriers can either lack an electron to form four covalent bonds (called an *acceptor impurity* or *P-doping*, because it accepts and holds on to an electron and moves a positive charge around) or have an electron left over after forming four covalent bonds with surrounding silicon atoms (called a *donor impurity* or *N-doping*, because it donates an electron and moves a negative charge around).

Aside from changing the conductivity/resistance, doping allows for control over the ability of current to flow from one direction to the other, as electrons from the N-doped zone occupy the directly neighboring holes in the P-doped zone, forming a depletion zone and blocking any further flow of current in that direction, which would only grow the depletion zone. This is also the principle behind the commonly used diode, which allows current to flow in only one direction.

When a voltage is applied to the gate, an electric field will form between it and the base (which is connected to the source/ground). This field will force charge carriers out of the way and move back the depletion zone, eventually forming a conductive zone between both N-doped zones, which in turn allows current to flow.

MOSFETs

By placing two N-doped areas on a larger P-doped substrate (Figure 1.1), a depletion layer forms between the N and P zones, preventing current from flowing as long as no voltage is applied to the gate in the case of an enhancement mode transistor which is used for this research. [2]

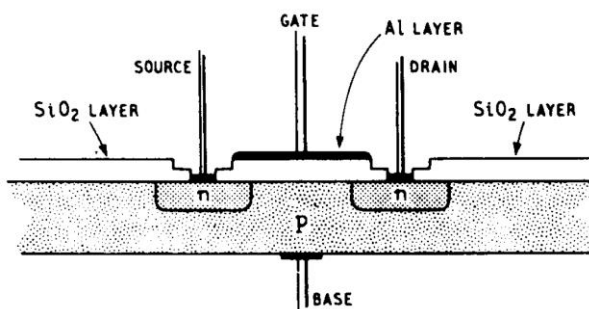


Figure 2.1: diagram of a mosfet. [2]

3. Variables

Gate-source voltage

As the voltage over the gate and the source increases (increasing the strength of the electric field), the conductive zone grows and increases in conductivity. This increase is almost linear once the threshold voltage is passed. [3]

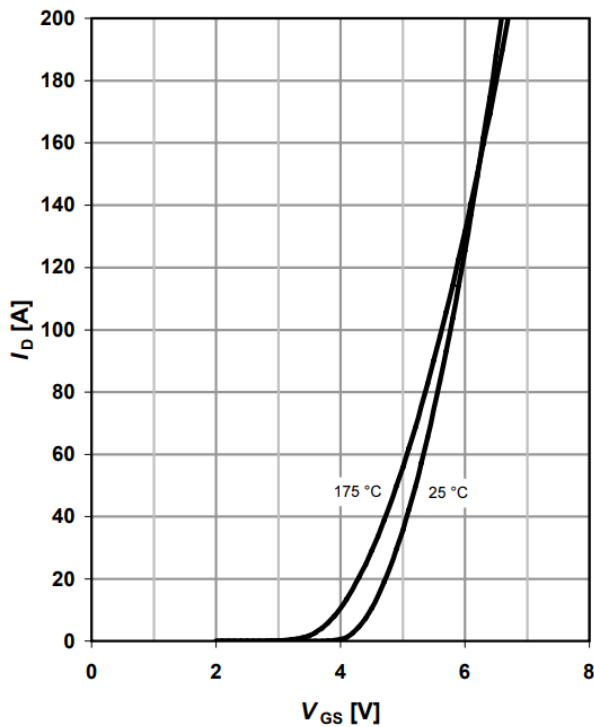


Figure 3.1: the increase in current flowing through a transistor as V_{GS} increases [3]

The relation between the conductivity and V_{GS} is initially quadratic, as shown between 4 and 5 V at 25 °C in figure Figure 3.1. This relationship can initially be characterized using equation (3.1), where k is a transistor- and temperature- dependent variable. [4]

$$I_{ds} = k * (V_{gs} - V_{th})^2 \quad (3.1)$$

At higher voltages, beyond the transition phase, this relationship becomes linear, eventually settling on a constant slope. At this point, the relationship can be characterized using equation (3.2), where

g_{fs} is the transconductance, which is the temperature- and V_{ds} -dependant slope of the I/V_{GS} -curve shown in Figure 3.1. [4]

$$I_{ds} = g_{fs} * (V_{gs} - V_{th}) \quad (3.2)$$

Temperature

Although semiconductors, of which silicon is one, show an inverse relation between temperature and resistance as the amount of charge carriers increases as covalent bonds thermally break down, as shown in Figure 3.2, semiconductors only provide a fraction of the conductivity in a mosfet. [2]

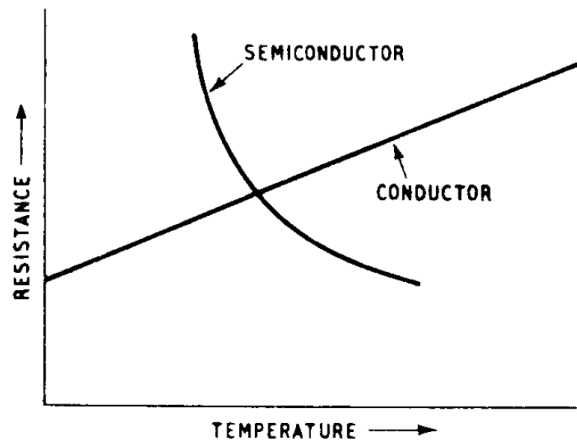


Figure 3.2: the relation between temperature and resistance for regular conductors and semiconductors. [2]

The vast majority of the conductivity is provided by the free charge carriers originating from the impurities added by doping. These impurities follow the common positive linear relation between temperature and resistance.

When doping the material and thus introducing the impurities, a concentration of merely 1 part in 10^8 is enough to cause a 16-fold increase in conductivity. Because of this, only a few parts in 10^8 are added. [2]

As such, only a tiny fraction of the conductivity is provided by the

semiconductor material. The effect of the inverse relation is negligible in comparison to the impurities. In practice, the R/T-diagram of a mosfet exhibits a slight upward curve, as can be seen in figure Figure 3.3.

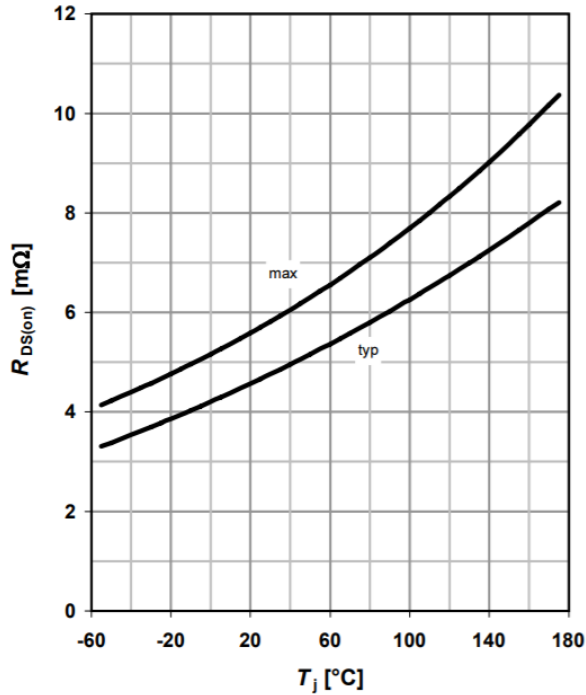


Figure 3.3: the relation between the source-drain resistance in the on-state and the temperature. [3]

The nature of this curve can be traced back to the fact that more temperature produces more kinetic energy in the material which can cancel out holes and electrons by scattering them, reducing the conductivity in the material. [5]

This relation can be described using equation (3.3), where $R_{25^{\circ}\text{C}}$ is the resistance at 25 °C and T_j is the junction temperature in K. The same relationship applies to equations (3.1) and (3.2), where k and g_{fs} change as the temperature changes. [4] [6]

$$R = R_{25^{\circ}\text{C}} * \left(\frac{T_j}{300}\right)^{2.3} \quad (3.3)$$

Drain-source voltage

The current, or the rate of change in charge over time, is dependent on the voltage and resistance of a circuit as described in Ohm's law [7]:

$$I = \frac{dQ}{dt} = \frac{V}{R} \quad (3.4)$$

where Q is the charge in coulombs (C), t the time in seconds (s), I the current in amperes (A), V the voltage in volts (V) and R the resistance in ohms (Ω).

The capacitance of a component, or the amount of charge that can be stored in said component, is given in farads (F), where each farad can store one coulomb per volt, which can be written as the formula

$$C = \frac{Q}{V} \quad (3.5)$$

or alternatively

$$V = \frac{Q}{C} \quad (3.6)$$

Assuming the charge Q at a given time t is equal to $Q(t)$, it can thus be said that

$$\frac{dQ}{dt} = \frac{Q(t)}{R * C} \quad (3.7)$$

which is a differential equation that can be solved by first turning the right-hand side into 1:

$$\frac{dQ}{dt} * \frac{R * C}{Q(t)} = 1 \quad (3.8)$$

followed by integration with respect to t :

$$\int \frac{dQ}{dt} * \frac{R * C}{Q(t)} dt = \int 1 dt \quad (3.9)$$

which can be worked out in two steps:

$$\int R * C * \frac{1}{Q(t)} dQ = t \quad (3.10)$$

followed by

$$R * C * \ln(Q) = t \quad (3.11)$$

Rewriting $C = \frac{Q}{V}$ to $Q = C * V$ and substituting gives

$$t = R * C * \ln(C * V) \quad (3.12)$$

Because t signifies the time it takes for a transistor to fully charge / discharge, we can relate this to the maximum switching frequency of a mosfet: the mosfet cannot fully switch before it is discharged.

It can thus be said that the relation between the peak frequency and the operating voltage is as follows:

$$f_{max} \propto \frac{1}{R_{ds} * C * \ln(C * V_{ds})} \quad (3.13)$$

where C is the capacitance across the depletion zone in the mosfet that needs to be filled given in farads and V_{ds} is the voltage applied over the drain and source in volts.

4. Thermodynamics

Heat generation

Although a perfect transistor would have infinite resistance while turned off (thus allowing absolutely no current) and infinite conductivity while turned on (thus experiencing no voltage drop over drain-source), practice shows transistors exhibit quite significant resistance while switching. This leads to a significant amount of power loss, which is released as heat in the junction.

As the junction makes up a mere 1.0% of the total mass of the transistor (Figure 4.1), which is only about 1.952 grams, little energy is required to significantly change the temperature of the junction.

For the junction, which is made out of silicon (0.01960 g), which in turn has a specific heat of $0.71 \frac{J}{g \cdot K}$, this comes down to $0.014 \frac{J}{K}$, or when working with continuous heat generation due to current, $71 \frac{K}{W \cdot s}$.

In reality, the junction will not nearly heat up this fast, as it is connected to a copper frame with 70x the mass of the junction, thus absorbing much of the heat.

On top of that, the dissipation to the environment will eventually match the amount of power generated within the junction, thus stabilizing the temperature.

TO-220 BOM 3							
Component	Material Name	Material Mass (g)	Element Name Composition	CAS #	Substance Mass (g)	Material Analysis Weight (%)	% of Total Weight
Chip	Silicon	0.01960	Si	7440-21-3	0.01960	100%	1.0%
			SiO ₂	7631-86-9	0.44041	84%	22.6%
Encapsulant	Epoxy Resin	0.52430	Epoxy	90598-46-2	0.03670	7%	1.9%
			Other	-	0.04718	9%	2.4%
			Cu	7440-50-8	1.37324	100%	70.4%
Lead Frame	Copper	1.37530	Sn	7440-31-5	0.00206	0%	0.1%
			Pb	7439-92-1	0.01098	95.5%	0.4%
			Sn	7440-31-5	0.00023	2%	0.1%
Die Attach	Soft Solder	0.01150	Ag	7440-22-4	0.00029	2.5%	0.1%
			Al	7429-90-5	0.00580	100%	0.3%
Wire Bond	Aluminum	0.00580					
Lead Finish	Matte Tin*	0.01520	Sn	7440-31-5	0.01520	100%	0.7%

Figure 4.1: the composition of a TO-220 BOM 3 MOSFET [8]

Heat flow

The heat that is generated within the junction will be dissipated to its environment, which is the case. The case will then dissipate the heat into the room. The rate at which this happens, can be described using the formula

$$\frac{\Delta Q}{\Delta t} = -KA \frac{\Delta T}{x} \quad (4.1)$$

where ΔQ is the change in energy, Δt the change in time, K a thermal conductivity factor, A the surface area, ΔT the temperature difference and x the distance. [9]

As $\frac{\Delta Q}{\Delta t}$, or the change in energy over the change in time (joules per second), is equal to the power loss over the transistor once thermal equilibrium is reached, it can simply be replaced by the power (P).

Also, $-\frac{KA}{x}$ depends only on the transistor, which does not change in size, material or surface area, and can thus be replaced by a single constant R_θ , which is the thermal resistance of the transistor.

This means the formula has been reduced to

$$P = R_\theta \Delta T \quad (4.2)$$

The junction temperature can be expressed using the case temperature by combining

$$P = c_1(T_j - T_a) \quad (4.3)$$

and the variant of the same formula, but for the case

$$P = c_2(T_c - T_a) \quad (4.4)$$

into one single formula

$$c_1(T_j - T_a) = c_2(T_c - T_a) \quad (4.5)$$

Creating a new constant $c = \frac{c_2}{c_1}$ and moving T_a to the right side of the equation finally yields

$$T_j = c(T_c - T_a) + T_a \quad (4.6)$$

This means the junction temperature can be calculated from the ambient temperature and the case temperature once the ratio between the thermal resistance of the junction to the air and the case to the air is known.

Method

In order to test the hypothesis, a total of three experiments were performed: two smaller experiments to measure the thermal characteristics of the transistor under load and one large experiment to measure the way the peak frequency as circumstances (T , V_{gs} and V_{ds}) change. This chapter describes the method used to obtain the results.

5. Equipment

Transistor

The transistor used was a single Infineon IPP057N06N3G (TO220-3 format), which is a power-transistor and often used in power circuitry to switch high currents and voltages. With its large capacity it is a good candidate for this study as the rise-time is well within the bounds we can measure.

Power supplies

Three bench power supplies were used to provide the necessary control over voltage across and current through the transistor.

A Delta ES 075-2 bench supply was used for the V_{gs} in measurements where a DC signal had to be provided to the gate to keep the transistor on for the whole measurement. The Delta ES 030-5 and ES 030-10 were used for V_{ds} , being able to provide currents of 5 and 10 A respectively.

The power supplies have the capability to be capped at varying voltages and currents, allowing precise control over the circumstances the transistor under which the transistor is put. The power supplies can also be connected in parallel, making it possible to reach a total current of 15 A, which is more than enough for our intents and purposes.

Measuring voltage and current

To measure the applied voltage, a digital multimeter (Keysight 34465A) was used, which can measure the voltage with an accuracy of a tenth of a millivolt, which in

turn is important when measuring the precise drain-source and gate-source voltages. The Voltcraft 860 and Tektronix DM250 were used to measure the voltage and current more precise than the power supply itself provided. The Voltcraft 860 was also used to measure applied power source voltage when the Keysight 34465A was in use to measure source-drain voltage.

Controlling Temperature

The temperature of the transistor was changed by placing it in a modified fridge. The fridge lacks a door and has a perspex pane installed in its place. In this window, there are multiple holes to access the inside of the fridge and to run cables through. At the bottom of the fridge, two incandescent light bulbs were placed with a combined power output of roughly 250 Watts, which could be lowered by using a knob on the plug.

To achieve high temperatures inside the fridge a generic heat gun was used to speed up the process. This was necessary as the heat loss by conduction through the fridge and pane, and airflow through holes greatly reduced the speed at which the temperature rose above 50°C.

A small fan was placed next to the transistor (but not pointed straight at or away from the transistor) to help promote air circulation within the fridge.

In the end, this allowed the ambient temperature to be varied between 20 °C and 70 °C.

Measuring temperature

The ambient temperature inside the fridge was measured approximately by an Ikea Fantast Meat Thermometer with its probe in the fridge, which was accurate enough to give an overall idea of current temperature range. The precise ambient temperature and the case temperature of the transistor were then measured by using an infrared camera (FLIR i60), which has an accuracy of 0.1 °C and can measure non-reflective surfaces up to 150 °C, as seen in Figure 6.1.

Producing signals

To apply AC signals to the gate, a Philips PM 5786 was used. This pulse generator has a rise-time of 2 nanoseconds and can generate square waves at frequencies ranging from 1 Hz to 125 MHz with controllable amplitudes up to 5 V.

While the signal itself was clean, channels A and B were not equal in amplitude, possibly due to the age of the function generator. Because of this, channel B was used to switch the transistor and its amplitude had to be measured separately each time the amplitude was changed by disconnecting it from the transistor, measuring it using the oscilloscope and reconnecting it.

Measuring the signal

To measure the frequencies, rise and fall times and the amplitude of the wave over the drain and source of the transistor a Tektronix TDS5104B oscilloscope was used. It has a sample rate of 5 GSA/s allowing it to measure up to 1 GHz.

Circuitry

The process was first tested on a simple PCB (as seen in Figure 5.1), which turned

out to distort the signal too much. Reflections caused ripples in the signal and the frequencies that could be attained were almost an order of magnitude below the expected values.

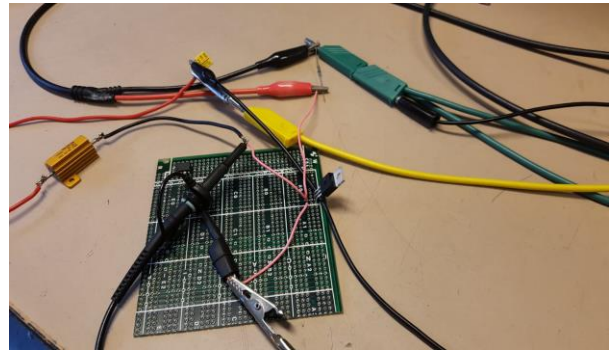


Figure 5.1: the primitive setup used during the initial phase

Through trial and error reflections were eventually traced to a few culprits.

The usage of a resistor containing a wound wire, essentially functioning as a spool, significantly lowered the peak frequency.

On top of that, each and every transition between two wires, be it soldered or using clips, caused reflections to occur. Proper connectors, like RF coaxial connectors, had a negligible effect. Soldered joints were also tolerable, but clips certainly had to be avoided and twisting two wires together to form a connection had a devastating effect on the signal.

Additionally, a proper ground plane would help to further reduce reflections.

It was thus decided that a custom task-specific PCB had to be designed and built to minimize all of these effects as much as possible.

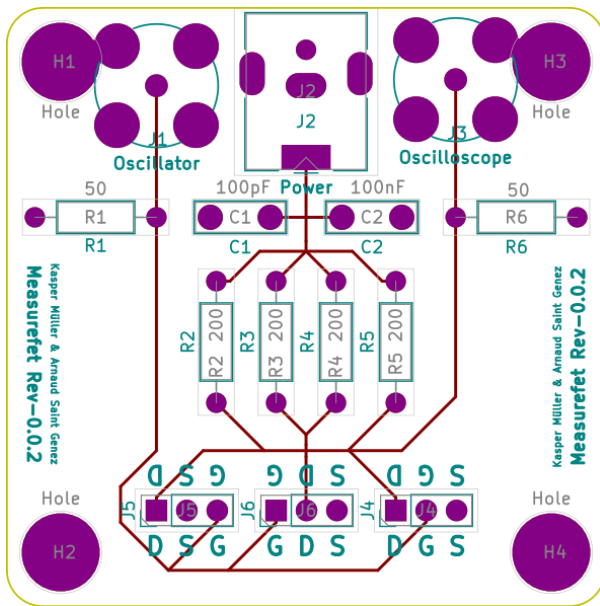


Figure 5.2: the finished PCB-layout for the schematic of the third experiment.

The design of the PCB can be seen in Figure 5.2 and the schematic can be found in Figure 7.3. The PCB was designed using KiCad and ordered from JLCPCB, after which the components were soldered on to the PCB, as can be seen in Figure 5.3.

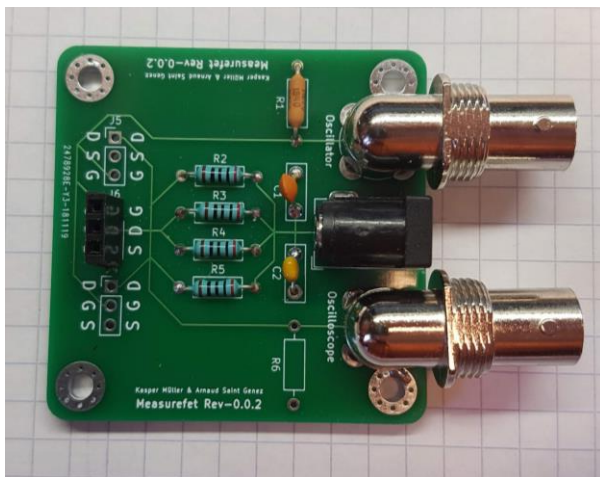


Figure 5.3: the physical PCB for the third experiment.

The 5x5 cm PCB is constructed with a ground plane on the back and short traces on the front. Coaxial cables can be used to plug in both oscillator and scope directly. On the left there is place for three 1x3 female header plugs to accommodate all pin layouts of typical mosfet cases.

Power is fed from a DC jack plug. C_1 and C_2 are capacitors to smooth out dips in voltage coming from the power supply (V_{ds}). R_1 is a resistor to smooth out reflections from the cable [10]. R_2 , R_3 , R_4 & R_5 are parallel metal film 200 Ω resistors. Together they form a current-limiting resistor of 50 Ω that can dissipate 1W of heat. Metal film resistors were used instead of wire wound or carbon resistors to minimize noise in the signal.

The PCB was used for the third experiment. The circuits for the first two experiments, which did not rely on the transistor switching on or off, could simply be constructed by soldering cables or clipping cables together.

6. Variables

Temperature

The incandescent lamps at the bottom of the fridge were used to heat up the inside of the fridge, which allowed us to place the transistor in environments ranging from 20 °C up to 70 °C.

To raise the temperature, the lamps were first turned to their highest power setting using the knob. If a high temperature (>50 °C) was desired, a heat gun was used to blow hot air through the hole in the pane.

The heat gun was used at a low power setting (roughly 90 °C) and aimed up or down in to the fridge to promote air circulation and to avoid directly blowing hot air on to the transistor.

Once the meat thermometer showed a temperature about 5 to 10 °C above the desired temperature, the heat gun was turned off.

The air in the fridge was given time to mix and reach equilibrium. The walls also still absorbed a lot of heat at this point, eventually leading the temperature to drop below the desired temperature.

Once the temperature dropped below the desired temperature, the heat gun was reintroduced to warm everything up to about 3 °C above the desired temperature.

At that point, the fridge was given time to equalize once more. By now the walls would have warmed up and the temperature would barely drop. The lamps, which were at full power the whole time, would be turned to about half power, or three quarters at higher temperatures, to keep up with the heat loss through dissipation.

The infrared camera was then used to measure the case temperature of a loose, unconnected transistor next to the live transistor. Once the temperature of this transistor had remained constant for about a minute, meaning thermal equilibrium had been reached, measurements could commence.

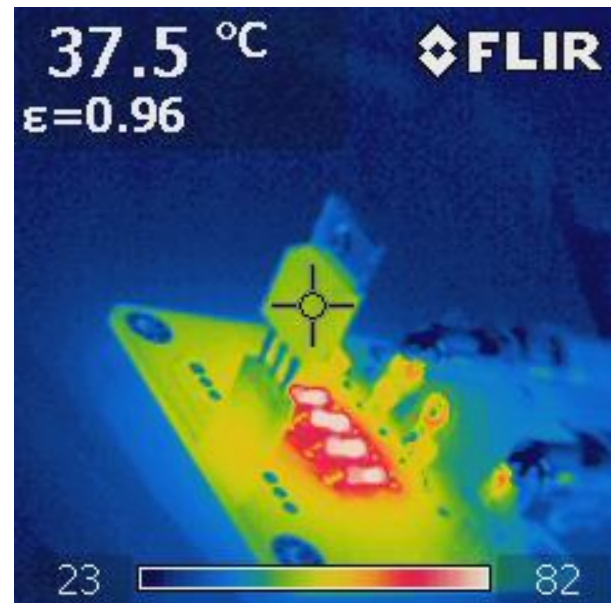


Figure 6.1: measuring the case temperature of the transistor.

Voltage

For the first two experiments, V_{ds} was measured by connecting two probes to the prongs on the transistor, which were plugged in to female headers, leaving a small part of the prong exposed. The probes could then directly measure V_{ds} at the base of the transistor.

During the first two experiments V_{gs} was measured by connecting a multimeter straight to the power supply.

For the third experiment, V_{ds} was measured by taking the amplitude of the wave that was measured by the oscilloscope.

During the third experiment, V_{gs} was measured by switching the coaxial cables on channels A and B on the waveform generator (with B previously being connected to the PCB and now being connected to the oscilloscope), after which the average high was taken from the oscilloscope. The offset on the waveform generator was set in such a way that the low was 0 V.

Resistance

For the first two experiments the resistance had to be measured.

As the transistor only has a resistance in the single-digit milliohms in its on-state, directly measuring the resistance with a simple multimeter is unreliable. The probes themselves comparatively also have a lot of resistance, not to mention the noise which alone already varied the measurement by multiple milliohms every second.

The resistance thus had to be measured by running a current through the transistor and measuring V_{ds} using the method previously described, which yields the voltage drop over the transistor. The voltage drop, combined with the current running through the transistor, could then be used to calculate the resistance using equation (3.4).

Using this method, the influence of the probes should be negligible, as the voltmeter should have an infinite resistance, or at least many orders of magnitude more than the transistor. As such, for all intents and purposes no current can run through it, meaning it does not influence the voltage drop.

Peak frequency

The exact definition of the peak frequency is quite subjective, so the most important part when measuring it is consistency.

Figure 6.2 shows the output signal of the transistor in yellow, with channel A of the waveform generator in blue. The transistor is operating slightly below its peak frequency.

Note the spikes that are visible in the output signal, which coincide with the cycle of the input signal. In Figure 6.2, this overshoot was manually offset to touch the gridline. Decreasing the frequency keeps the spike exactly on the gridline, never crossing it.

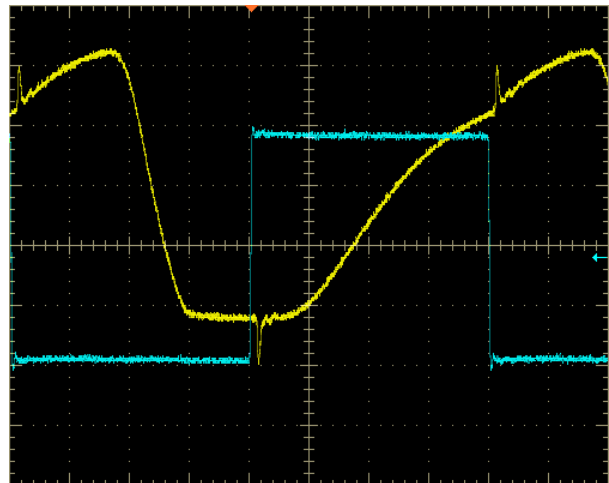


Figure 6.2: the input & output signals (blue/yellow) while under f_{max} .

Increasing the frequency, on the other hand, would eventually make the spike move up as the amplitude of the wave collapses. The spike was found to be the most reliable and consistent indicator by a large margin.

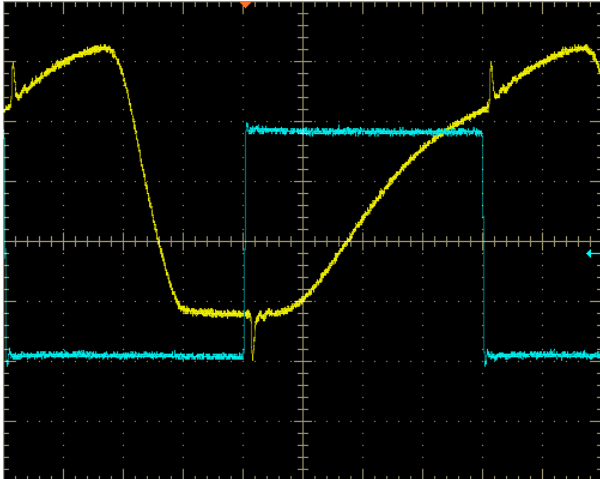


Figure 6.3: the input and output signals (b/y) while at f_{max} .

In Figure 6.3, this overshoot moved up half a minor tick mark, which was the definition used for f_{max} , as this indicates the amplitude is collapsing.

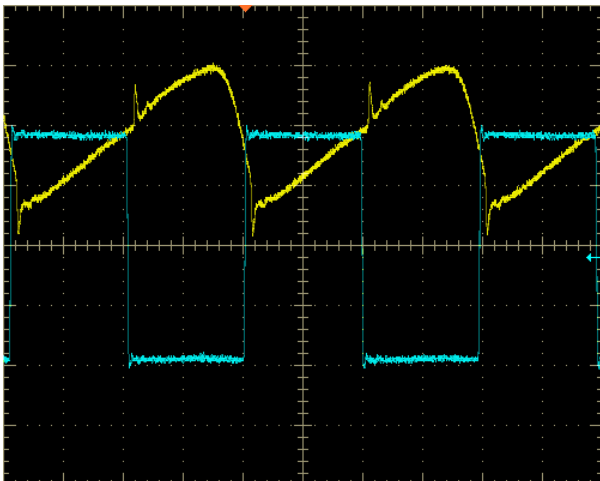


Figure 6.4: the input and output signals (b/y) past f_{max} .

Figure 6.4 shows the by now collapsed amplitude. Where the transistor could previously still fully turn on and off, it now fails to reach 0 V before beginning its next cycle. In other words, the transistor is not capable of completing a full cycle at this frequency, thus operating past its peak frequency.

7. Experiments

Relation between R_j & T_j

The first experiment compares the resistance of the transistor in its on-state, with as little current flowing through it as possible, to the ambient temperature.

The transistor is placed in the heating fridge and connected as described in Figure 7.1. R_1 is a resistor of $10\ \Omega$ to limit the current and capable of dissipating $10\ \text{W}$.

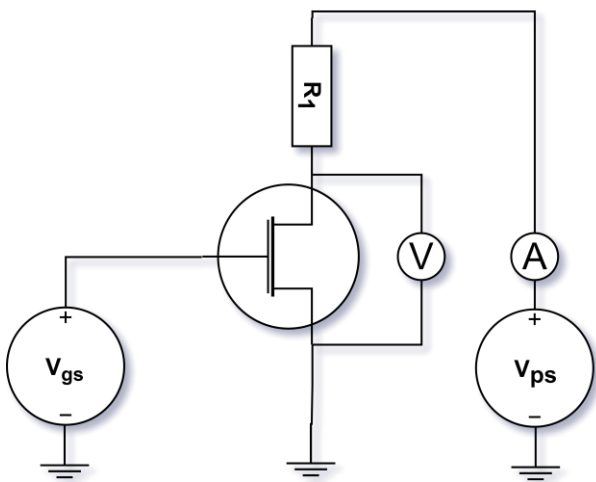


Figure 7.1: circuit diagram for measuring the R_j of an open transistor.

V_{gs} is set to a constant voltage of $5\ \text{V}$, which is a bit more than $1\ \text{V}$ above V_{th} , but still relatively low, leading to a relatively high resistance, which in turn leads to more accurate results.

V_{ps} is set to a constant voltage of $10\ \text{V}$ at the power supply, although this will almost fully be lost over the resistor. In the end, the voltage drop over the drain and source (V_{ds}) will only be a couple of millivolts.

The temperature inside the fridge is increased in steps of about $5\ \text{K}$. Once the desired temperature is reached, T_c is given a minute or two to become equal to T_a , after which the measurement begins.

This is done by turning on both power supplies and measuring the voltage across the drain source in addition to the current through the transistor. These values can be used to calculate the resistance of the transistor.

The power sources are turned off again and the process is repeated until the maximum temperature inside the fridge ($70\ ^\circ\text{C}$) is reached.

The measurement is time-sensitive, as the junction will warm up when $1\ \text{A}$ ($10\ \text{V}$ over $10\ \Omega$) is flowing through it. The rate at which this happens is in the neighborhood of $0.4\ \text{K}$ per second as calculated on page 15. As such, measuring the voltage drop in less than five seconds is desirable.

Relation between T_a & T_j

The second experiment compares the temperature difference between the junction and ambient to the temperature difference between the case and ambient.

The transistor is placed outside the fridge in a large room with a stable ambient temperature and connected as shown in Figure 7.2.

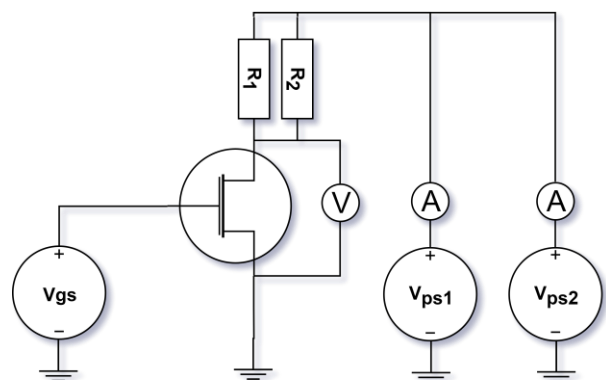


Figure 7.2: circuit diagram for measuring T_c under load.

R_1 and R_2 are two parallel resistors of 0.5Ω each, together acting as a single resistor of 0.25Ω capable of dissipating 20 watts.

Exactly the same voltage as the first experiment is applied to V_{gs} (5 V). The voltage remaining untouched is a very important detail, as a small change in V_{gs} leads to large changes in the resistance, while small changes in resistance are relied on to calculate T_j . For this reason, the settings of the power supply (V_{gs}) should not be changed between experiments, as there is little chance of being able to configure it to exactly the same settings as before, which is crucial for calculating T_j .

The two power supplies (V_{ps1} & V_{ps2}) are turned on while being limited to 1 V and 0 A.

The limit of the power supplies on the current is then raised until the sum of the currents is equal to about 1 A.

The transistor is given about three minutes to warm up, after which both the ambient temperature (once again with a disconnected transistor) and the case temperature of the transistor are measured. On top of that, the sum of the currents and the voltage drop over the source and drain is measured.

The limit on the current is then raised once more, this time to 2 A, after which the transistor is once again given time to heat up before T_a , T_c , I_{ds} and V_{ds} are measured.

This process is repeated up to and including 12 A, at which point the transistor is starting to hit its thermal limits.

Relation of V_{gs} , V_{ds} , T_j & f_{max}

The third experiment measures f_{max} under a wide range of circumstances in an attempt to reveal any patterns.

For the last experiment the transistor is placed in the female headers on the custom PCB, which forms the circuit seen in Figure 7.3.

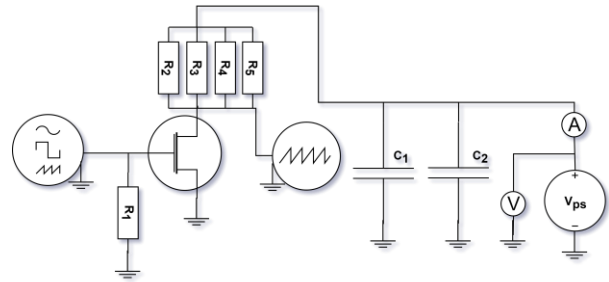


Figure 7.3: circuit diagram for measuring f_{max} under set conditions.

V_{gs} is set on the waveform generator using the method previously described, after which the PCB is connected to channel B, giving full control over V_{gs} and f .

The power supply is also plugged in to the DC-jack on the PCB, giving control over V_{ds} . Due to the resistors, the voltage applied at the power supply V_{ps} does not match the actual V_{ds} , which will have to be measured at the transistor.

The oscilloscope is connected to the waveform generator and the PCB, allowing f_{max} and V_{ds} to be measured.

The PCB is then placed in to the fridge in its entirety, also giving control over T_a and thus also T_j . A disconnected transistor is also placed next to the live one, both of which can be seen through the access hole, allowing the temperature to be measured using the IR camera.

At this point T_j , V_{ds} , V_{gs} and f can all be measured and changed, allowing

measurements into the influence of the former three on the latter.

The transistor is turned on and is made to switch close to its peak frequency. When the case temperature of the transistor is stable, the precise f_{max} is measured as previously described under

Peak frequency. The amplitude of the output signal (V_{ds}), the amplitude of the input signal (V_{gs}), the current through the resistor (I_{ds}), the temperature of the disconnected transistor (T_a) and the temperature of the transistor's case (T_c) are all recorded. Additionally, the signal's rise and fall time are also written down.

Measurements can be done with all kinds of values for each variable within the specifications of the equipment at hand. These measurements can be performed completely at random as they all contribute to the large dataset that can be analyzed later to find the relations.

In practice, the variables were set semi-randomly. That is to say, preference was given to certain values. For example, at

most temperatures a measurement with a V_{ds} of 8 V at the power supply was performed. On top of that, while many variables were randomly set, they were generally set to whole numbers or fractions.

This was done to obtain some sets of data where only one variable changed to examine the influence of that variable only on the transistor.

Truly stable variables can, however, never be obtained, as changes in V_{ds} will also have a significant impact on T_j , which increases as V_{ds} increases. T_a can also not be kept completely stable, as the fridge has a large heat capacity and takes a long time to respond to temperature changes. This means precisely matching the heat loss is practically impossible, especially within a reasonable amount of time.

For this reason, a different approach was taken where as large of a data set as possible is collected, which can then be analyzed without spending copious amounts of time meticulously managing the temperature, etc.

Results & Analysis

While the full data set can be found in the appendix, this chapter is dedicated to analyzing the data and visualizing measurements that were performed to be able to draw a conclusion afterwards.

8. Thermal characteristics

R over T_a

The first experiment was performed with the intention of deriving the relation between the temperature of the junction (T_j) and resistance (R).

Using equation (3.4) on the data from Table 13.3, R (with $R = V_{ds}/I_{ds}$) can be plotted against T_j (with $T_j = T_a$), shown in Figure 8.1.

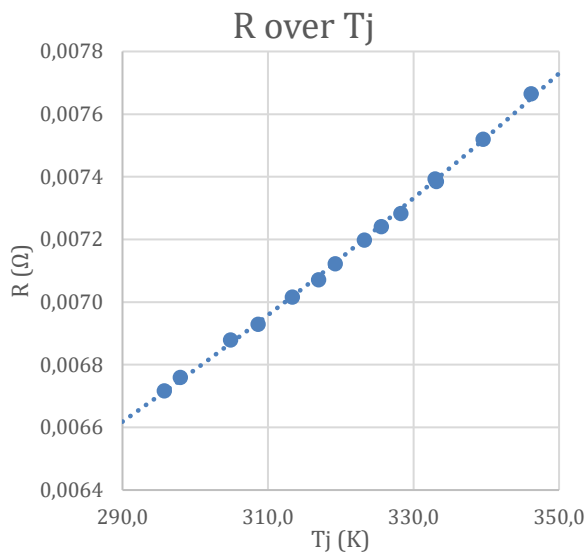


Figure 8.1: the resistance of the open gate ($V_{gs} = 5 V$) compared to the temperature of the gate.

The trendline shown was obtained through regression analysis. By taking equation (3.3) and adding an offset b , yielding the general formula $R = a(T/300)^{2.3} + b$, and trying out a wide range of values for a and b until the sum of the differences between the measured values and the calculated values squared is as low as possible, a trendline can be found.

This was quickly done using a simplified version of Figure 14.1 with only 2 variables, giving $a = 0.002221$ and $b = 0.004564$, which yields the trendline in equation (8.1).

The average variation² using these values is $6.303 * 10^{-11}$.

The offset b had to be added because there was no other way to get the trendline to run through the data points, as a straight line drawn through the data already crosses the y-axis above the x-axis, which means there is no parabolic function without an offset that runs through all points.

Next, the equation

$$R = 0.002221 \left(\frac{T_j}{300} \right)^{2.3} + 0.004564 \quad (8.1)$$

can be rewritten to

$$T_j = 4273.6(R - 0.004564)^{10/23} \quad (8.2)$$

This formula allows the conversion of the resistance to the temperature, although it only holds true for the precise V_{gs} used to find the values in Table 13.3.

$(T_j - T_a)$ over $(T_c - T_a)$

The second experiment was performed to calculate the ratio between the differences in temperature between the junction and ambient, and the case and ambient.

The second experiment was performed at the same V_{gs} as the first one, so equation (8.2) holds true for this experiment.

Using the data from Table 13.4, R can once again be calculated using equation (3.4). This value for R can then be used to calculate T_j using equation (8.2).

Combining the now known T_j with the measured values for T_c and T_a allows $(T_j - T_a)$ to be plotted over $(T_c - T_a)$, which can be seen in Figure 8.2.

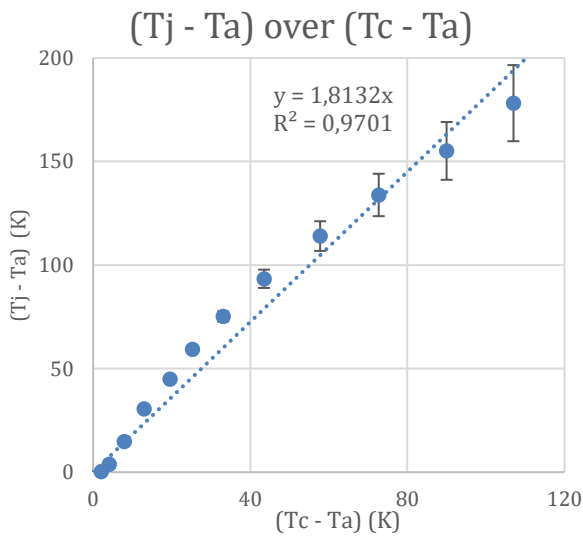


Figure 8.2: the difference in temperature between the junction and ambient compared to the difference between the case and ambient.

The error bars are added to indicate the uncertainty that arises from the extrapolation using equation (8.2), as the regression was performed on temperature measurements ranging from roughly 300 to 350 K, while it is used to calculate temperatures up to about 470 K. The datasheet of the transistor affirms the trend up to about 450 K, which means the

extrapolation is not unwarranted. Small variances in the values for a and b used to draw the trendline do, however, cause differences in the calculated T_j .

To find this variance, a slightly modified version of the code with a third variable c was used to find a trendline in the form of $R = a(T/300)^b + c$, which yielded the values 0.001572, 3.0838 and 0.005217 for, respectively, a , b and c , yielding equation (8.3) once solved for T_j .

$$R = 2433.4(T_j - 0.005217)^{0.3243} \quad (8.3)$$

The average variation² was $2.845 * 10^{-11}$, which is quite a bit lower than the previous $6.303 * 10^{-11}$ for equation (8.2).

The difference between the value for R given by equation (8.2) and equation (8.3) was used as the error bar.

Equation (4.5) can also be written in the form of equation (8.4), which shows the slope of Figure 8.2 is expected to be a constant, which is the same constant c used in equation (4.6).

$$\frac{T_j - T_a}{T_c - T_a} = \frac{c_2}{c_1} = c \quad (8.4)$$

The linear trendline (of the form $y = ax$) given by Excel in Figure 8.2 shows c to be equal to 1.8132, which means equation (4.6) can now be written as:

$$T_j = 1.8132(T_c - T_a) + T_a \quad (8.5)$$

From now on, T_j can thus be calculated using equation (8.5).

9. Electrical characteristics

f_{max} over T_j

Table 13.9 contains measurements that were performed at constant values for V_{ds} and V_{gs} , which means only T_a and thus also T_j significantly vary. Using equation (8.5) to calculate T_j , Figure 9.1 can be drawn.

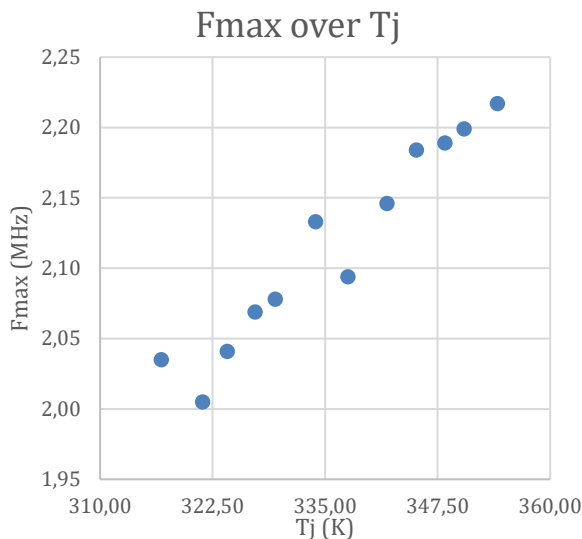


Figure 9.1: the peak frequency compared to the junction temperature with $V_{gs} = 5\text{ V}$ and $V_{ds} = 4\text{ V}$.

Clearly, a positive relation between the junction temperature and peak frequency can be seen. Although it seems to be roughly linear, the temperature range is too small and the standard deviation is too large to accurately draw a trendline without further knowledge about the mechanisms behind this relation.

f_{max} over V_{ds}

In a similar fashion Table 13.13, Table 13.14, Table 13.15, Table 13.16 and Table 13.17 can be used to compare the peak frequency to V_{ds} .

Once again using equation (8.5) to calculate T_j , the peak frequencies can be plotted against the temperature for the varying temperature ranges, yielding Figure 9.2.

It should be noted that T_j varies by about 10-15K within each table, even causing some overlap between the tables. The temperatures should thus not be interpreted too literally, as they are rounded off significantly.

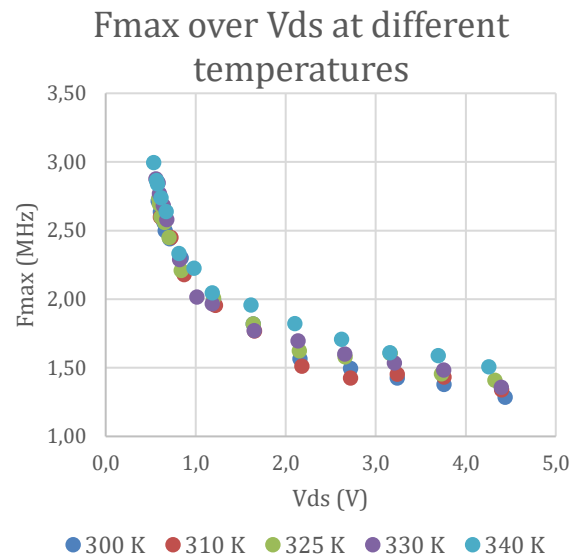


Figure 9.2: the peak frequency compared to the drain-source voltage at different ambient temperature ranges with $V_{gs} = 4,67\text{ V}$. Note the temperatures given are approximations and actual values vary by about 8 K in each direction.

The trend closely matches the general shape of $y = 1/\ln(x)$, which was the expected trend. Attempts to draw a trendline through the data will be made in a different section.

Additionally, the positive relation between temperature and f_{max} , as seen in Figure 9.1, is also visible in Figure 9.2, where higher temperature ranges generally reach higher frequencies for the same V_{ds} , even when taking the variance in temperature into account.

f_{max} over V_{gs}

V_{gs} is the only varying variable in *Table 13.12*, which allows T_j (using equation (8.5)) to be plotted against V_{gs} , yielding Figure 9.3.

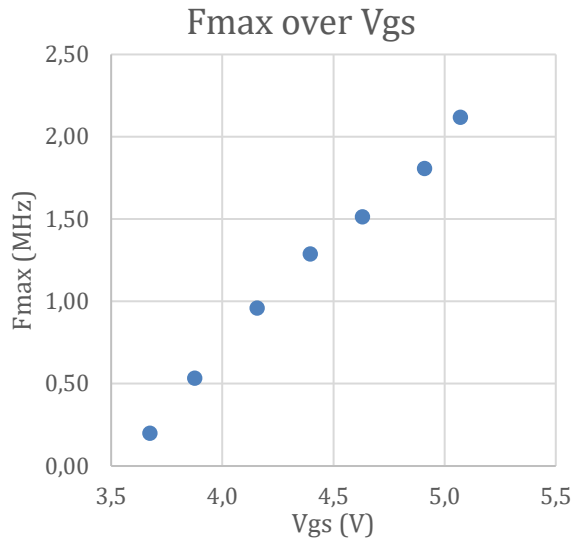


Figure 9.3: the peak frequency compared to the gate-source voltage with $V_{ds} = 8\text{ V}$.

Although the voltage range is not very large, the relation does seem to be largely linear.

The relation does not seem to start out as a parabola as hypothesized, as the first measurement was taken very near to V_{th} , at which point the slope is already seemingly constant, ruling out a parabolic trend.

Figure 3.1 also rules out the possibility of the range being too small, as the parabolic trajectory of I_{ds} over V_{gs} ends around $V_{gs} = 5\text{ V}$, which coincides with the range of the measurements.

The two results that were left out could not fully switch states, which means these two measurements cannot be compared to the others.

10. Regression analysis

To analyze the full data set and draw a trendline that used all three variables (V_{gs} , V_{ds} and T_j), regression analysis was performed. This was done using Python 3.7, the code for which can be found in Figure 14.1.

Models

Three models were tested. The first (equation (10.1)) being the hypothesis with some offsets added.

The second model (equation (10.2)) was the same as the first, but with the original variable b permanently being set to 3.1, based on multiple test results from the first model returning values for b around 3.1, and a new exponent on the temperature (which originally was 2.3) being set to variable b .

The third model (equation (10.3)) was a combination of the first and second, where the original variable b remained and a new variable f was added to find the optimal exponent for the temperature, but run at a lower resolution, as the time required to finish increases exponentially with the amount of variables.

$$f_{max} = \frac{a}{\left(\left(\frac{T_j}{300}\right)^{2.3} + c\right) * \ln(d * V_{ds} + e)} \quad (10.1)$$

$$f_{max} = \frac{a}{\left(\left(\frac{T_j}{300}\right)^b + c\right) * \ln(d * V_{ds} + e)} \quad (10.2)$$

$$f_{max} = \frac{a}{\left(\left(\frac{T_j}{300}\right)^f + c\right) * \ln(d * V_{ds} + e)} \quad (10.3)$$

All three models were tried both with $n=1$ and $n=2$.

Results and graphs

The outputs and settings used for each model can be found in Table 13.18.

Although the regression was run using all data (except when noted next to the data in the appendix), the following graphs were drawn only using the data from table Table 13.7 up to and including Table 13.17, but excluding Table 13.12, as they contain the vast majority of the measurements but exclude all 'exotic' values for V_{gs} , simplifying the drawn graphs enormously.

The regression formula was specified in the Matlab program and drawn using the code found in

Figure 14.2, which was then plotted in GNU Octave.

The zoomed out graph of the trendline found by model 1 with $n=2$ (Figure 10.1), drawn using three commonly used values for V_{gs} during the measurements, seemingly shows a decent fit to the data.

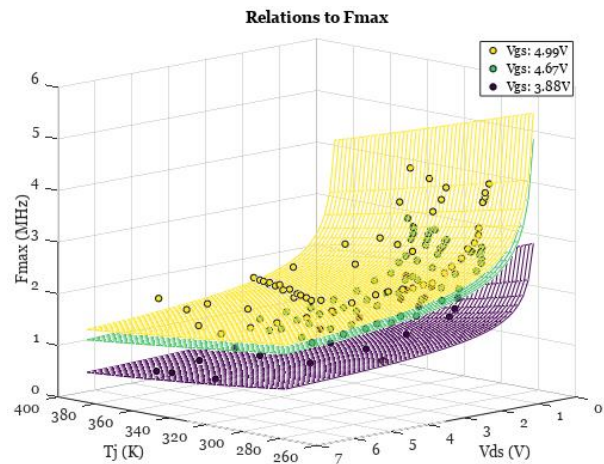


Figure 10.1: the trendline of model 1 ($n=2$) zoomed out.

The average var^2 was 0.06563, which shows room for improvement, but also means the model is not very far off on average.

More interesting is the graph viewed from the side, as done in Figure 10.2 and Figure 10.3. This reveals more about the accuracy of the model.

The thick line represents the value of the trendline at the point of view, while the shaded zones indicate the values farther back (keeping in mind this is a 3D graph viewed from one side). All yellow points should thus, ideally, be in the yellow area and all green points should be in the green area, etc.

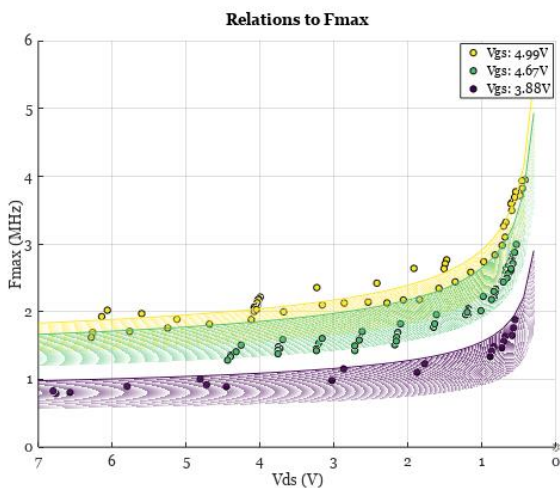


Figure 10.2: the trendline given by model 1 ($n=2$) looked at from the V_{ds} -side.

Figure 10.2 shows the model used for V_{ds} is quite accurate, as the shape of the trendline almost precisely matches the line upon which the data points lie.

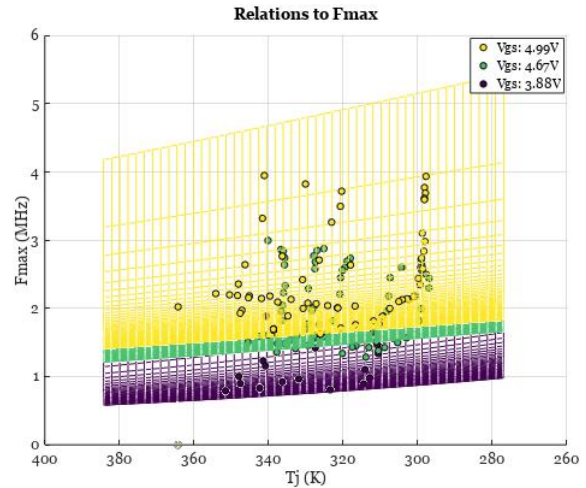


Figure 10.3: the trendline given by model 1 ($n=2$) looked at from the T_j -side.

Figure 10.3, on the other hand, shows that the model used for T_j does not match up with the measurements. As model 1 was based on the hypothesis which predicted a negative relation between the temperature and peak frequency, which was already disproven in Figure 9.1, this is to be expected.

For this reason model 2 and 3 were tested.

Table 13.18 shows model 2 (0.02928 var^2) to be a bit more accurate than model 3 (0.03425 var^2), and significantly more accurate than model 1 (0.06563 var^2). For model 2 and 3 the linear model for V_{gs} outperformed the quadratic model in both cases, which is in line with the conclusions of Figure 9.3.

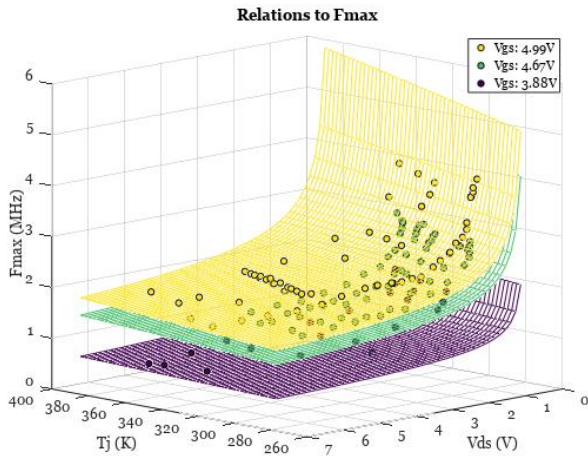


Figure 10.4: the trendline of model 2 ($n=1$) zoomed out.

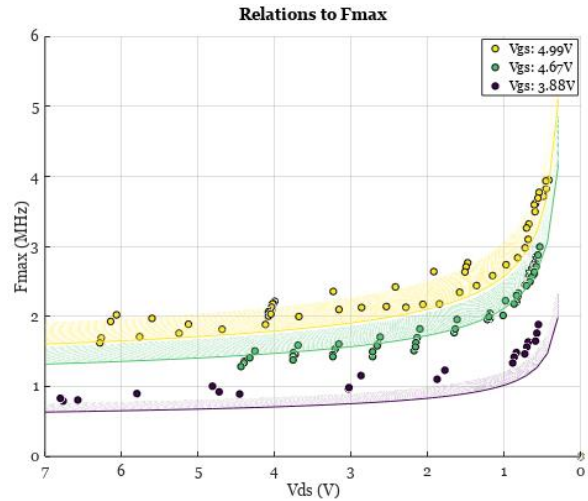


Figure 10.6: the trendline given by model 2 ($n=1$) looked at from the V_{ds} -side.

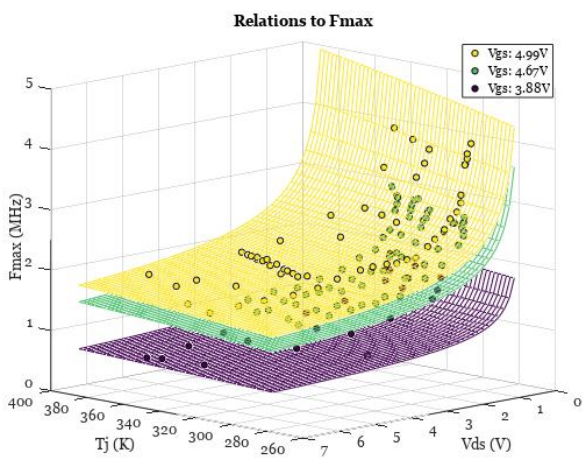


Figure 10.5: the trendline of model 3 ($n=1$) zoomed out.

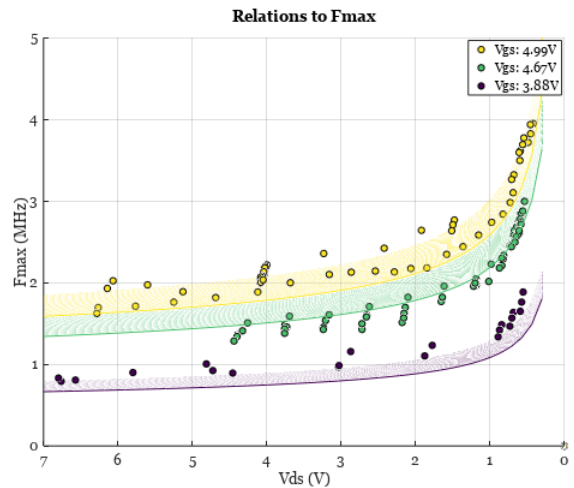


Figure 10.7: the trendline given by model 3 ($n=1$) looked at from the V_{ds} -side.

Figure 10.4 and Figure 10.5 show a lot of similarity, which should be expected as the values of their variables are all very close to each other, as seen in Table 13.18.

Figure 10.6 and Figure 10.7 logically show the same level of similarity. The fact that the datapoints for $V_{gs} = 3.88$ do not lie within the purple zone is remarkable, but still entirely possible as the area of purple zone has become smaller when compared to Figure 10.2, meaning the square of the variance can still be smaller without lying in the zone.

The most important graphs for the temperature, Figure 10.8 and Figure 10.9, viewing the trendlines of model 2 and 3 from the T_j -side, show the models now actually roughly matching the relationship between T_j and f_{max} .

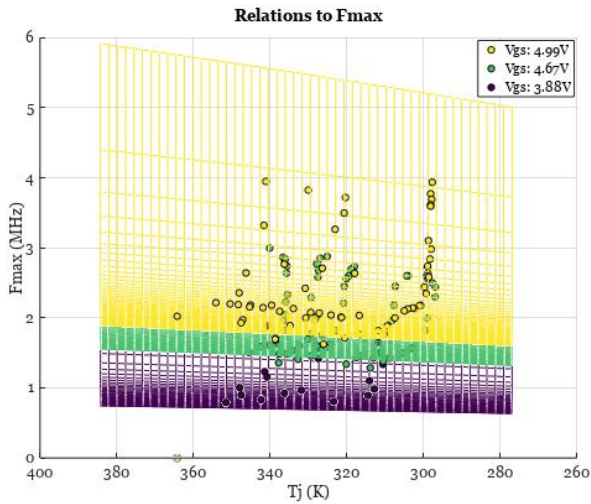


Figure 10.8: the trendline given by model 2 ($n=1$) looked at from the T_j -side.

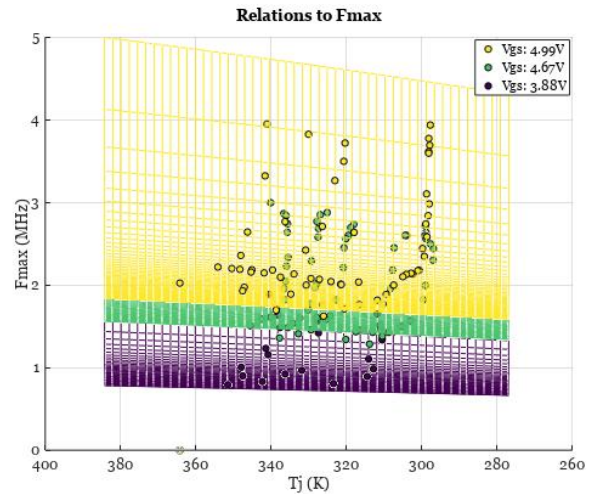


Figure 10.9: the trendline given by model 3 ($n=1$) looked at from the T_j -side.

All in all, model 2 with the values from Table 13.18 ($n=1$) is a representative model for the influence of V_{gs} , V_{ds} and T_j on f_{max} .

Conclusion

11. Summing it up

There is strong correlation between the junction temperature, the drain-source voltage, the gate-source voltage and the highest attainable frequency of a mosfet, all of whom influence the peak frequency to a large extent. Equation (11.1) provides one of many possible models to represent this relation, with an average value of 0.02928 for the square of the difference between the modeled f_{max} and the measured value, indicating the accuracy of the model and thus the fact that the peak frequency is influenced by T_j , V_{ds} and V_{gs} to a large extent.

$$f_{max} = \frac{4.565}{\left(\frac{\left(\frac{T_j}{300} \right)^{-0.476}}{(V_{gs} - 3.1)^2} - 0.0326 \right) * \ln(35.666 * V_{ds} - 4.866)} \quad (11.1)$$

Discussion

This section discusses some anomalies that were encountered, the limits of the research conducted and the validity of the conclusion.

12. Hypothesis & conclusion

Hypothesis

The hypothesis was partially correct.

The hypothesized relationship between V_{ds} and f_{max} closely matched the observed relationship, with an inverse logarithm of the drain-source voltage only requiring a few offsets to almost perfectly match the data points.

A relation similar to the hypothesized relation between V_{gs} and f_{max} was observed, although the hypothesis stated this would initially be quadratic and then straighten out in to a linear function, which was not observed in practice, where the relationship was about linear from the beginning. A larger range of voltages is, however, needed before a linear relation can be concluded with certainty. Further research could thus be conducted by using more advanced waveform generators capable of generating square waves with amplitudes of 10 or more volt, although the required frequencies will quickly become very high if the current observations are to be trusted, especially at lower values for V_{ds} , which means the research might have to be restricted to V_{gs} only, with a relatively high (>4 V) value for V_{ds} .

The hypothesized relation between the junction temperature and the peak frequency was incorrect, as a negative relation was expected, while a positive relation was observed. This could possibly be explained by the fact that V_{th} decreases as the temperature increases, as can be seen in figure Figure 12.1. As the influence

of V_{gs} was found to be rather large, the positive influence of T_j on $(V_{gs} - V_{th})$ by lowering V_{th} probably outweighs the influence of T_j on the resistance.

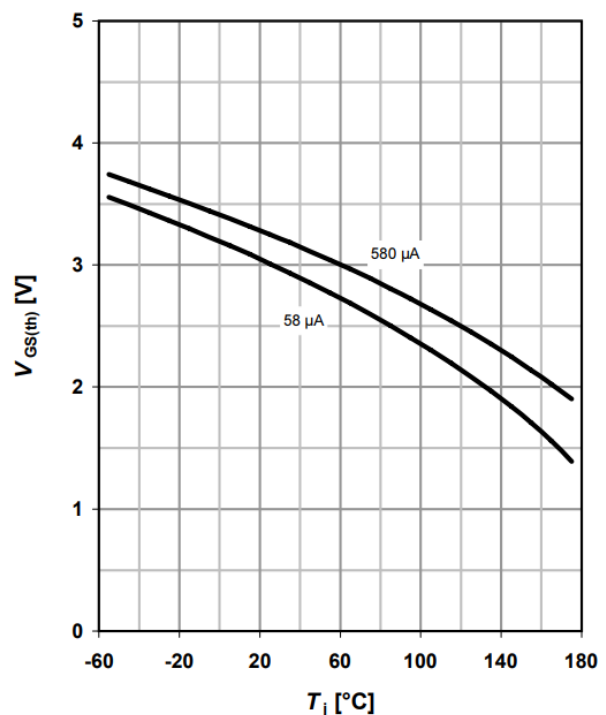


Figure 12.1: the relation between T_j and V_{th} . [3]

Further research on this topic could be conducted by conducting a similar experiment to this one, but looking at the relation between T_j , V_{th} , V_{gs} and f_{max} . This could be combined with the in-depth look at V_{gs} that was previously mentioned.

Remarkable observations

A couple of remarkable observations were made.

The first being the requirement of an offset to accurately describe the relationship

between the temperature and resistance. The theory stated this should not be the case, as equation (3.3) shows.

A possible explanation for this observation could be the additional resistance of other components in the transistor, like the prongs, of which a small part was included in the measured value. The slope of the measured values roughly matches the slope of the theoretical values provided by Figure 3.3, further supporting this hypothesis, although the observed values were higher. It should be noted that Figure 3.3 was measured with a V_{gs} of 10 V, which further lowers the resistance, in part explaining the discrepancy. Although you would expect the slope to drop with the higher V_{gs} , which it does not seem to do.

To further research this topic, measurements on a wider range of temperatures would have to be performed. The fridge used for this research physically could not handle temperatures above 80 C, as the paint started forming bubbles and the perspex pane started expanding to the point where it could no longer fit.

One solution would be to use a stationary heat gun, with the transistor close to the nozzle, as this allows temperatures north of 200 °C to be reached while requiring no enclosure, which far exceeds the specifications of the transistor and is thus the highest one can possibly go.

When using a heat gun, care should be taken to make sure the transistor is actually at a constant temperature, which means the transistor cannot be heated up, then left to cool while performing measurements along the way, which was initially tried instead of the fridge, but yielded inaccurate results as a temperature difference between the case and the junction starts to form the second the

external heat source is remove and the case starts to cool.

Performing measurements at lower temperatures, for example by using liquid nitrogen, is also important, as this reveals whether the offset that was introduced to solve the trendline continues to hold true.

To check whether the hypothesis of the prongs being responsible for the offset is right, a transistor would need to be disassembled to be able to place the probes closer to the junction. The resistance is measured using the voltage drop, which means only the distance between the two contact points for both probes matters. The influence of the probes themselves is negligible in comparison to the large resistance of the voltmeter.

Another remarkable observation was the non-linear curve in Figure 8.2. The theory states this should be constant.

Another way of approaching this issue is by taking the thermal resistance R_{θ} , given by equation (12.1) [6]. Using the data from Table 13.4 and equation (8.5), this resistance can be graphed against the power for the case and junction (where the calculation of the case uses T_c instead of T_j).

$$R_{\theta} = \frac{\Delta T}{P} = \frac{T_j - T_a}{I_{ds} * V_{ds}} \quad (12.1)$$

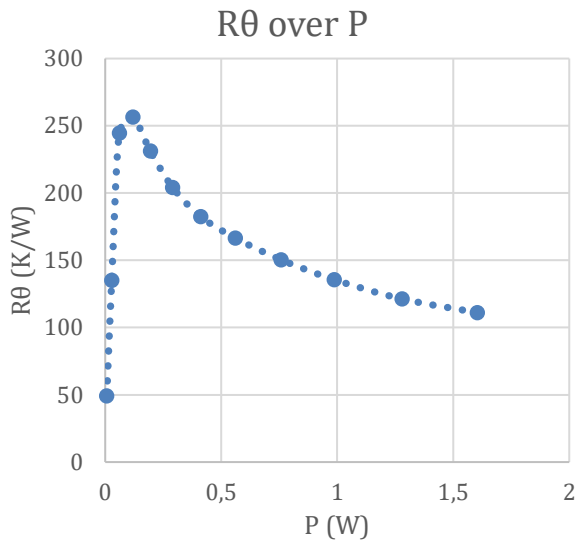


Figure 12.2: the thermal resistance of the junction to ambient over the heat generated per second

This relation should be constant, as the junction, case and ambient air do not physically change aside from the temperature. The fact that the values for low amounts of power are off is not that surprising and can easily be explained by the fact that small errors in the measured values for T_a and T_c can easily mean ΔT becomes 0.2 instead of 0.1 (an error of merely 0.1, which was the accuracy of the IR camera), doubling the calculated value for the thermal resistance.

For larger amounts of power, however, these measurement errors become less relevant as ΔT grows. It would thus be expected for the points in the graph to converge to a constant thermal resistance. This does seem to happen, as the slope gradually keeps on dropping as the power increases, but even at temperatures in excess of 100 °C for the case and significantly more for the junction, this relation does not become constant.

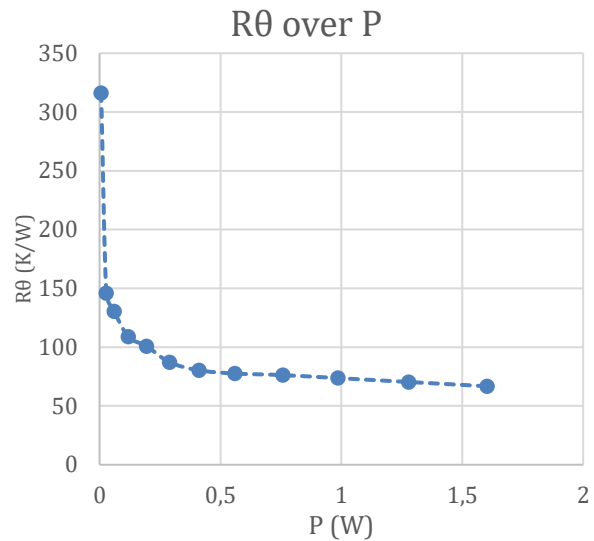


Figure 12.3: the thermal resistance of the case to ambient over power

Figure 12.3 shows the same graph, except for T_c instead of T_j (so the thermal resistance of the case to ambient). This figure shows precisely what was expected: large variations at lower amounts of power as measurement errors dominate, but a constant value as the difference in temperature grows and the errors become insignificant.

This suggests there is a more deeply rooted issue at hand, possibly relating to the way T_j is measured. As the values for T_j are calculated using an extrapolation of other measurements, multiple smaller inaccuracies might add up to a larger error in equation (8.2), which could in turn be responsible for the slope in Figure 8.2 and Figure 12.2, which is why the error bars were added to the former of the two. The error bars do not, however, fully explain the difference, as it still remains impossible to draw a straight line through them.

Further research would be needed to solve this problem, possibly by measuring the resistance at a wider range of temperatures as previously mentioned.

For this research, it means the value of c used to form equation (8.5) might be off by roughly 0.3 to 0.5 in either direction. This does not mean the observed positive relation between T_j and f_{max} can suddenly become negative, although it can change the slope of the relation.

Regression analysis

The form of analysis used bring several problems with it.

The way the regression works is that it tries several values for one or more variables, called a and b for example, in a wide range. An example of this would be starting at -100, then trying all values up to and including 100 in steps of 10, so -100, -90, -80, 90, 100. It does this for a and then also for b at every tried value of a , which means it has to try a total of $21 * 21 = 441$ combinations. For every variable added, the amount of combinations that has to be tried is multiplied by 21. With the 12 steps in each direction and 5 variables used for this research, every step takes $25^5 = 9765625$ attempts. Out of these attempts, it then picks the one with the lowest sum of the squares between the calculated and measured values and then proceeds to try new values in a narrower range. So if the optimal value for a in a range of -100 to 100 with steps of 10 was 30, it would now try values between 20 and 40 in steps of 1. If 28 was optimal, it would try values between 27 and 29 in steps of 0.1, and so on.

What this means, is that adding a single variable or slightly increasing the resolution comes at a large cost. Increasing the resolution to 15 steps in both directions and adding a single variable brings the previous amount of tries from roughly 10 million to 31^6 , which is almost 900

million. Running the regression already took 4 to 5 hours to complete for every single run, which would increase to about 400 hours with the new variable and slight increase in resolution.

This is not practical, so a relatively low resolution with as few variables as possible has to be used.

The effect of fewer variables is clear: the models cannot be too complex.

The effect of the resolution, on the other hand, is less clear. Although the number of decimals calculated can easily be increased by increasing the amount of times the program zooms in (which scales linearly, compared to the amount of steps which increase parabolically and the amount of variables, which increases exponential, so $run\ time \propto zoom\ count * steps^{variables}$), this does not solve the problems that arise from the lower amount of steps / the increases step size.

The value of the variance² does not have a single low-point and can have valleys when plotted. As the lowest value is taken and all other values are discarded, the risk of taking a sub-optimal value and discarding the optimal value rises as the resolution decreases.

This can happen if, for example, $a = 30$ has a higher variance than $a = 55$, but a lower variance than $a = 50$ and $a = 60$. If steps of 10 are taken, 55 is skipped and 30, 50 and 60 are all tried. As 30 yielded the lower variance, it is taken and all values between 50 and 60 are discarded. Doubling the resolution to steps of 5 would have solved this problem, as 55 would have been tried and yielded a lower variance.

By looking at the data it can be confirmed that this does indeed happen, as model 2,

which had less variables than model 3 and could thus use a higher resolution, outperformed model 3, which was exactly the same as model 2 but with an extra variable to replace a static value in model 2.

This value was manually set to 3.1 in model 2 and, through regression, came turned out to be 3.193 in model 3.

Model 2 was disadvantaged, as it could not change the value to a better one to get a lower variance. If 3.11 was better than 3.1, model 3 had the ability to change this. The models are identical in every other way.

Yet model 2 still outperformed model 3 by yielding a lower variance, which means model 3 must have skipped over the optimal value and discarded it.

It must thus be understood that the model given in the conclusion is merely *a* model that happens to yield a low variance, not *the* model that absolutely matches the observed values.

Although this problem can never be fully solved, it can be reduced up to the point where it is negligible by increasing the resolution. Multithreading the calculation is a must at that point, in addition to rewriting the program in a more efficient language than Python, like C or C++, or by using Python libraries that are written in a more efficient language. Not using lists to store values and possibly even simplifying the model to reduce it to 4 variables would help further reduce the required amount of time for a single run.

This problem was a major factor in the decision to include all data in the appendix.

Method

From the fact that a single coaxial cable, normally capable of perfectly transmitting signals at far higher frequencies than the transistor can ever reach, could influence the results as much as it did (compare f_{max} in *Table 13.7* and *Table 13.8*, identical in every way except for the cable), only because it had a different impedance, it can be concluded that even the circuit used to test the transistor most definitely has massive influence on the results. The resistors, waveform generator and oscilloscope all had an impedance of 50 Ω . Additionally, using different cables with the same impedance had no effect on the result, so no impedance mismatch should be present in the results used for the conclusion.

As previously mentioned, this was also the deciding factor in the decision to make the custom PCB for the experiments, as there was no other way to ensure high quality connections between components. This was an effective move, as switching from the soldered parts to the PCB brought a large increase in f_{max} with it.

For further research, this is an important factor to look at.

One more factor to be conscious of, is the fact that the PCB as a whole was placed in to the fridge, which means the resistors also changed in temperature. As their resistance increases with temperature, their share in the voltage drop increases. This can be seen in the data (*Table 13.9*), as V_{ds} drops by roughly 1% over the course of the experiment with the increasing temperature, while V_{ps} remains constant.

At roughly 4 V for V_{ds} , this 1% change is far too small to explain the 10% difference in f_{max} (see the slope at 4 V in *Figure 9.2*), but still important to be aware of.

The last problem was also a factor in the decision to use multi-dimensional regression analysis, which self-corrects for this kind of problem.

As the peak frequency increased, it seems like the difference in resistance is too small to have an effect similar to the cable with a different impedance.

Validity of the conclusion

Wherever possible care was taken to ensure the results of the experiments were valid, even if that meant starting back at zero and as such, the largest problems have been solved. Some problems do, however, remain. These have previously been described.

The influence of these relatively small issues is debatable. They certainly do have influence on the results, but the amount is uncertain.

Even when parts were soldered together and the output signal was rife with reflections, the transistor exhibited the same characteristics as with the higher quality circuits, albeit at a significantly lower peak frequencies.

As the quality of the circuit improved with each iteration, the signal became clearer and the peak frequency rose. When the custom PCB was eventually assembled, which is at the limits of what was possible with the available time and resources, the

results were consistent and of high certainty.

This means that, while the peak frequency could possibly be raised slightly higher, the results are not expected to be any different.

On top of that, *Table 13.8* and *Table 13.11* show that the rise and fall time depend heavily on V_{gs} and V_{ds} . When taking this in to consideration, the specifications of the transistor state the rise and fall time to be, respectively, 68 and 9 nanoseconds at $V_{gs} = 10\text{ V}$ and $V_{ds} = 30\text{ V}$. This is roughly in line with the observed rise and fall times (taking in to account the fact that the higher source-drain voltage will cause them to be higher, but the higher gate-source voltage will cause them to be lower again, thus ending up in the same order of magnitude), indicating the observations using the circuit are not too far off from the specifications.

All in all, the trends in the measurements are rather valid, although no conclusions surrounding the temperature can be drawn except for the fact that the relation is positive. V_{ds} and V_{gs} are more certain, although debate is always welcome, so anyone is welcome to disprove them.

The models themselves only serve to show the correlation for this specific transistor, not as definitive formulas for any transistor.

Reflection

Many things were learned while performing this research. This chapter contains some of our personal experiences.

Experiences

The most frustrating aspect of the research was the amount of times most things had to be redone, generally due to an error on our part. This was the #1 time sink during the whole project. Most parts of this document have been rewritten multiple times, either due to poor word choice / vagueness, or due to new information requiring large scale changes in the project. For example, the relation between temperature and resistance was first said to be linear, which in hindsight was a rather naïve assumption. Because the measured values also looked linear, this error went unnoticed until Mirjam kindly pointed it out. At that point, the theory, hypothesis, regression model, excel sheets and graphs all had to be redone.

To make matters worse, we passed over Figure 3.3 multiple times in the specsheet, so it could have easily been avoided by double-checking. This goes to show making claims that are only supported 75% should not be done and can have grave consequences.

It also demonstrates the value of having a third party double check the work, as these errors, once made, generally go unnoticed by the person who made them, even when reading over them again, whereas a third party might instantly see something is wrong.

Another example was the cable with a different impedance that was used. Most cables had an impedance of 50Ω , so after at some point they were just mindlessly grabbed without checking the impedance.

This resulted in the usage of an unmarked cable with an unknown impedance, which was only noticed when a different cable was used later on, requiring the earlier measurements to be redone and thus taking another day.

Similar setbacks occurred at almost every step of the process, even when everything seemed to be going well, misfortune struck when most unexpected, like the day we successfully spent measuring the thermal properties of the first transistor, only to blow it up by sending something in the neighborhood of 1 KW through it, causing a massive spark and a dead transistor literally minutes before INCAA closed, requiring *everything* we did that day to be redone the next day.

The share of documentation in the amount of work during the process was also underestimated. Although the large amount of research that had to be done and the immense amount of text that had to be written was rather unsurprising, the difficulty of properly maintaining a >50 page document and collaborating turned out to be far greater than anticipated. Initially, Google Docs was used to document everything, but this was later changed to Microsoft Word for the more advanced formatting tools.

Although these tools are far more advanced, this also means more time has to be dedicated to making use of them. On top of that, working in parallel can be rather inconvenient, especially when having to merge two documents while

keeping the previously mentioned advanced formatting intact.

As a result of this, the last hours before a deadline were not spent improving or expanding the document, but formatting it and combining two versions. On top of that, it also meant only one person could do this, as the second person had to sit on the sidelines, wasting time while waiting for the documents to be combined. This is a major aspect where improvements could be made. Both by using the tools for collaboration provided by MS Word and by keeping this in mind when initially allocating time, which was not done as the time required was severely underestimated.

But not everything went horribly wrong. We were very fortunate to receive aid from as many people as we did. Their contributions have had a massive positive effect on this report and made working on it significantly more enjoyable.

We were also fortunate with our research setup. While we, for example, were busy struggling with a heatgun to warm the transistor, Robert kindly pointed us to the

modified fridge which just happened to be above Kasper's desk.

On top of that, all problems we encountered, however big they seemed, could eventually be solved, showing persistence does eventually pay off.

The success of the experiments was also a pleasant surprise, as we personally did not expect there to be as much correlation as there turned out to be and certainly did not expect the results to be as clear as they were.

Last but not least, we would be lying if we did not admit we profited massively from the unexpected extension of the deadline. Even though the last few days were 100% dedicated to working on the project, it shows there was more room for improvement as far as time management goes.

All in all, the project was a long journey with its ups and downs, but in the end we can honestly say we are happy with how it went, how much we have learned and how the end result turned out.

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Appendix

13. Raw data

On the following pages the raw data from our measurements, including all discarded results, can be found. Not included are results from the trial & error phase. The data is divided in to parts for clarity with comments wherever applicable.

Physical measurements

V_{gs} (V) V_{ps} (V) f_{max} (MHz) V_{ds} (V) T_a (°C) T_c (°C) I_{ds} (A) t_{rise} (ns) t_{fall} (ns)

Table 13.1: V_{ds} & I_{ds} with varying T_a (first measurement, first run)

Note: unused due to blowing the transistor up

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
5.032	-	-	0.005928	-	21.7	0.944	-	-
5.032	-	-	0.005981	-	26.1	0.941	-	-
5.032	-	-	0.006060	-	30.9	0.942	-	-
5.032	-	-	0.006145	-	36.0	0.944	-	-
5.032	-	-	0.006176	-	39.6	0.940	-	-
5.032	-	-	0.006282	-	43.8	0.942	-	-
5.032	-	-	0.006381	-	47.3	0.946	-	-
5.032	-	-	0.006476	-	52.8	0.946	-	-
5.032	-	-	0.006559	-	56.2	0.946	-	-
5.032	-	-	0.006624	-	61.6	0.940	-	-
5.032	-	-	0.006719	-	65.7	0.942	-	-

Table 13.2: V_{ds} , T_a & T_c with varying I_{ds} (second measurement, first run)

Note: unused due to blowing the transistor up and T_a measured using a thermostat

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
5.032	-	-	0.012646	24	28.3	2.044	-	-
5.032	-	-	0.019145	24	29.8	3.042	-	-
5.032	-	-	0.025872	24	32.5	4.056	-	-
5.032	-	-	0.049758	24	51.7	7.112	-	-
5.032	-	-	0.059426	24	62.2	8.110	-	-
5.032	-	-	0.069450	24	70.7	9.118	-	-
5.032	-	-	0.081500	24	85.6	10.150	-	-
5.032	-	-	0.094150	24	97.0	11.174	-	-
5.032	-	-	0.110562	24	112	12.206	-	-

Table 13.3: V_{ds}/I_{ds} with varying T_a (first measurement, second run)

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
5.072	-	-	0.006562	-	22.6	0.977	-	-
5.072	-	-	0.006777	-	35.5	0.978	-	-
5.072	-	-	0.006909	-	43.8	0.977	-	-
5.072	-	-	0.007032	-	50.1	0.977	-	-
5.072	-	-	0.007123	-	55.1	0.978	-	-
5.072	-	-	0.007215	-	60.0	0.977	-	-
5.072	-	-	0.007496	-	73.0	0.978	-	-
5.072	-	-	0.007354	-	66.4	0.978	-	-

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
5.072	-	-	0.007230	-	59.8	0.978	-	-
5.072	-	-	0.007082	-	52.4	0.978	-	-
5.072	-	-	0.006966	-	46.1	0.978	-	-
5.072	-	-	0.006862	-	40.2	0.978	-	-
5.072	-	-	0.006735	-	31.7	0.979	-	-
5.072	-	-	0.006611	-	24.8	0.978	-	-

Table 13.4: V_{ds} , T_a & T_c with varying I_{ds} (second measurement, second run)
 Note: T_a was measured using a thermostat, not the IR-camera

5.072	-	-	0.006675	22	24.1	0.995	-	-
5.072	-	-	0.013791	22	26.1	2.038	-	-
5.072	-	-	0.020541	22	29.9	2.952	-	-
5.072	-	-	0.029421	22	35.0	4.059	-	-
5.072	-	-	0.038314	23	42.6	5.076	-	-
5.072	-	-	0.047725	23	48.3	6.084	-	-
5.072	-	-	0.058113	23	56.1	7.094	-	-
5.072	-	-	0.069494	23	66.5	8.068	-	-
5.072	-	-	0.083173	23	80.8	9.116	-	-
5.072	-	-	0.097580	23	95.7	10.114	-	-
5.072	-	-	0.114474	23	113	11.174	-	-
5.072	-	-	0.132400	23	130	12.108	-	-

Table 13.5: f_{max} , V_{ds} , I_{ds} & T_c with varying V_{gs} , V_{ps} & T_a (third measurement, first run)
 Note: t_{rise} and t_{fall} were not measured, only V_{ps} was manually changed

3.995	12.002	0.688	6.680	24.0	37.6	0.146	n/a	n/a
3.995	10.997	0.661	6.140	24.1	35.0	0.134	n/a	n/a
3.995	10.006	0.690	5.640	23.9	32.7	0.119	n/a	n/a
3.995	8.996	0.740	5.120	23.7	31.1	0.109	n/a	n/a
3.995	8.000	0.770	4.560	23.7	30.7	0.098	n/a	n/a
3.995	7.499	0.806	4.360	23.3	29.9	0.090	n/a	n/a
3.995	6.999	0.810	4.060	23.2	28.9	0.080	n/a	n/a
3.995	6.504	0.850	3.720	23.1	28.6	0.075	n/a	n/a
3.995	6.007	0.878	3.480	23.3	28.6	0.073	n/a	n/a
3.995	5.504	0.886	3.400	23.5	28.3	0.068	n/a	n/a
3.995	5.004	0.892	3.120	23.5	27.9	0.062	n/a	n/a
3.995	4.498	0.928	2.760	23.3	27.3	0.052	n/a	n/a
3.995	3.996	0.938	2.480	23.7	26.8	0.046	n/a	n/a
3.995	3.503	0.973	2.180	23.6	26.3	0.042	n/a	n/a
3.995	3.006	1.012	1.860	23.5	25.9	0.037	n/a	n/a
3.995	2.505	1.059	1.560	23.7	26.0	0.032	n/a	n/a
3.995	1.999	1.078	1.360	23.5	25.3	0.020	n/a	n/a
3.995	1.503	1.104	1.120	23.2	24.7	0.018	n/a	n/a
3.995	1.006	1.188	0.980	23.3	24.8	0.014	n/a	n/a
3.995	0.707	1.289	0.830	23.7	24.2	0.008	n/a	n/a
3.995	0.501	1.367	0.760	23.2	24.3	0.006	n/a	n/a

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
3.995	0.403	1.415	0.730	23.4	24.5	0.005	n/a	n/a
3.995	0.302	1.524	0.680	23.6	24.5	0.003	n/a	n/a
3.995	0.206	1.545	0.680	23.6	24.2	0.002	n/a	n/a
3.995	0.100	1.635	0.640	23.8	24.3	0.000	n/a	n/a
3.995	0.000	1.682	0.610	24.0	24.2	0.000	n/a	n/a

Table 13.6: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, second run)
 Note: unused due to cable impedance mismatch, significantly influencing the frequency
 Only V_{ps} was manually changed

3.635	0.000	0.952	0.768	23.7	23.6	0.000	312	261
3.635	0.103	0.948	0.808	23.3	23.7	0.000	326	252
3.635	0.201	0.928	0.856	23.5	24.0	0.002	354	265
3.635	0.297	0.921	0.896	23.6	24.3	0.004	365	275
3.635	0.405	0.905	0.936	23.8	24.3	0.004	375	276
3.635	0.500	0.894	0.936	23.8	24.4	0.004	382	296
3.635	0.703	0.880	1.024	23.9	24.8	0.006	350	247
3.635	1.001	0.869	1.104	23.7	24.6	0.008	381	257
3.635	1.500	0.801	1.148	24.1	24.6	0.016	477	333
3.635	2.000	0.674	1.120	24.1	25.1	0.022	259	346
3.635	2.504	0.571	1.260	24.4	25.3	0.030	244	380
3.635	3.000	0.527	1.480	23.8	25.1	0.035	212	408
3.635	3.504	0.486	1.660	24.0	25.6	0.044	203	413
3.635	3.998	0.419	2.140	24.1	26.6	0.046	214	522
3.635	4.500	0.401	2.360	23.7	26.6	0.055	200	527
3.635	4.999	0.357	2.280	23.7	27.0	0.058	122	321
3.635	5.505	0.332	2.880	23.9	27.7	0.072	210	669
3.635	5.999	0.314	3.120	23.8	27.9	0.073	220	717
3.635	7.000	0.251	6.440	24.6	30.1	0.088	161	860
3.635	8.001	0.175	3.960	23.8	30.6	0.106	159	1211
3.635	9.000	0.167	4.480	23.7	32.0	0.118	160	1360
3.635	9.993	0.170	4.960	23.9	34.4	0.130	156	1420
3.635	10.998	0.162	5.480	23.9	36.3	0.144	150	1403
3.635	12.009	0.157	5.880	23.9	37.5	0.158	142	1423

Table 13.7: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, third run)
 Note: unused due to cable impedance mismatch, significantly influencing the frequency
 Only V_{ps} was manually changed

4.990	0.000	1.785	0.692	22.6	23.8	0.000	157	115
4.990	0.101	1.682	0.728	22.9	24.0	0.000	186	125
4.990	0.200	1.627	0.752	23.2	24.1	0.000	204	130
4.990	0.299	1.571	0.764	23.0	24.1	0.004	216	140
4.990	0.399	1.507	0.808	23.0	24.2	0.004	236	145
4.990	0.500	1.469	0.840	23.5	24.6	0.005	250	151
4.990	0.703	1.431	0.888	23.6	24.3	0.012	271	104
4.990	1.001	1.291	0.998	23.6	24.4	0.013	317	140

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
4.990	1.499	1.100	1.220	23.6	24.8	0.018	386	207
4.990	1.250	1.180	1.104	23.6	24.3	0.012	350	173
4.990	1.999	1.045	1.440	23.3	24.5	0.022	340	199
4.990	2.500	1.005	1.720	23.7	24.9	0.028	375	221
4.990	3.000	0.969	1.980	23.9	25.1	0.040	403	230
4.990	3.499	0.924	2.280	23.5	25.8	0.046	433	252
4.990	4.006	0.871	2.500	24.0	25.9	0.048	362	275
4.990	4.502	0.847	2.680	24.2	26.7	0.053	284	282
4.990	4.999	0.828	2.920	23.9	26.3	0.059	275	311
4.990	5.498	0.800	3.120	24.0	27.2	0.064	254	361
4.990	6.001	0.752	3.340	23.9	27.9	0.073	237	420
4.990	7.000	0.671	3.840	23.9	28.9	0.084	212	501
4.990	8.000	0.653	4.360	23.9	30.1	0.098	204	580
4.990	8.999	0.635	4.920	23.4	31.3	0.113	210	620
4.990	9.994	0.583	5.440	23.7	33.5	0.122	213	772
4.990	11.002	0.562	6.240	23.4	35.2	0.136	135	436
4.990	12.003	0.554	6.480	24.3	38.9	0.152	130	417

Table 13.8: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, fourth run)

Note: only V_{ps} was manually changed

4.990	0.000	3.936	0.456	22.6	23.7	0.001	63	13
4.990	0.099	3.773	0.544	23.0	24.1	0.002	66	15
4.990	0.201	3.691	0.560	23.1	24.0	0.004	70	19
4.990	0.302	3.615	0.592	23.2	24.2	0.005	75	22
4.990	0.403	3.595	0.608	23.2	24.2	0.006	78	23
4.990	0.499	3.102	0.688	23.4	24.6	0.008	86	29
4.990	0.702	2.981	0.728	23.6	24.3	0.011	85	35
4.990	1.002	2.839	0.825	23.7	24.5	0.016	96	43
4.990	1.503	2.739	0.976	23.6	24.8	0.022	131	46
4.990	1.998	2.583	1.152	23.3	24.5	0.028	156	50
4.990	2.500	2.441	1.362	23.9	25.4	0.036	173	52
4.990	2.995	2.346	1.583	24.0	25.2	0.042	191	54
4.990	3.499	2.180	1.847	23.5	25.7	0.048	200	58
4.990	3.991	2.173	2.064	24.1	26.2	0.054	170	60
4.990	4.499	2.131	2.285	24.0	26.9	0.062	173	63
4.990	5.004	2.142	2.540	23.7	27.0	0.070	183	66
4.990	5.501	2.128	2.864	23.9	27.8	0.075	208	70
4.990	5.996	2.100	3.160	23.9	28.3	0.084	227	71
4.990	7.001	1.999	3.683	23.8	29.6	0.096	233	74
4.990	7.998	1.885	4.120	23.8	30.7	0.110	208	73
4.990	9.010	1.817	4.686	23.6	31.9	0.124	226	79
4.990	1.000	1.761	5.249	23.9	34.7	0.136	235	81
4.990	1.100	1.711	5.764	23.3	36.6	0.152	235	82
4.990	1.199	1.623	6.283	24.3	40.1	0.166	241	84

V_{gs} (V) V_{ps} (V) f_{max} (MHz) V_{ds} (V) T_a (°C) T_c (°C) I_{ds} (A) t_{rise} (ns) t_{fall} (ns)

Table 13.9: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, fifth run)

Note: only T_a was manually changed

4.990	8.011	2.035	4.050	20.6	33.3	0.106	206	76
4.990	8.013	2.005	4.080	28.3	39.3	0.108	211	80
4.990	8.014	2.069	4.080	33.2	44.7	0.108	200	77
4.990	8.014	2.041	4.080	36.1	44.3	0.108	202	76
4.990	8.014	2.078	4.060	41.6	49.7	0.108	198	77
4.990	8.015	2.133	4.040	45.2	53.8	0.108	193	75
4.990	8.015	2.094	4.040	50.6	58.2	0.108	196	75
4.990	8.015	2.146	4.030	54.4	62.3	0.110	190	74
4.990	8.015	2.184	4.020	58.2	65.8	0.110	185	72
4.990	8.015	2.189	4.030	61.0	68.8	0.110	187	73
4.990	8.015	2.199	4.020	63.7	71.2	0.110	182	72
4.990	8.016	2.217	4.000	67.4	74.9	0.110	182	72

Table 13.10: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, sixth run)

Note: only V_{ps} and T_a were manually changed

4.990	11.990	2.023	6.066	65.9	79.8	0.163	195	80
4.990	4.002	2.641	1.921	65.7	69.8	0.058	175	55
4.990	0.998	3.321	0.679	66.3	67.5	0.016	73	35
4.990	0.000	3.948	0.418	65.6	66.9	0.000	61	14
4.990	6.988	2.357	3.234	62.6	69.4	0.096	166	70
4.990	8.001	2.156	4.028	59.5	66.5	0.110	186	74
4.990	2.997	2.767	1.478	57.9	60.8	0.044	164	51
4.990	0.000	3.824	0.454	56.2	56.6	0.000	65	10
4.990	11.001	1.973	5.602	53.5	64.8	0.150	204	82
4.990	12.002	1.929	6.143	54.3	65.4	0.163	206	84
4.990	8.002	2.181	4.025	50.7	59.4	0.109	188	73
4.990	5.000	2.422	2.422	49.9	54.2	0.070	156	63
4.990	2.999	2.709	1.499	48.9	51.3	0.043	172	52
4.990	1.002	3.264	0.709	48.1	49.1	0.016	73	30
4.990	0.500	3.497	0.598	47.2	47.4	0.080	71	17
4.990	0.000	3.718	0.487	46.5	46.9	0.000	65	14
4.990	8.004	1.999	4.085	46.5	52.6	0.110	205	76
4.990	10.002	1.889	5.126	45.5	54.4	0.136	214	82
4.990	12.001	1.695	6.265	44.7	56.2	0.164	232	88
4.990	3.002	2.636	1.514	42.1	43.6	0.042	178	55

Table 13.11: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, seventh run)

Note: only V_{ps} and T_a were manually changed

3.875	3.002	1.103	1.877	39.5	40.3	0.036	350	170
3.875	0.000	1.649	0.592	38.7	38.7	0.000	135	135
3.875	0.500	1.466	0.732	38.4	38.0	0.006	235	90
3.875	1.001	1.336	0.890	38.1	37.7	0.012	297	118
3.875	5.001	0.984	3.029	36.4	38.2	0.060	338	215

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
3.875	8.001	0.892	4.457	35.7	38.8	0.096	231	313
3.875	12.001	0.807	6.571	35.8	43.8	0.146	193	439
3.875	0.000	1.762	0.574	51.1	52.3	0.000	115	63
3.875	0.500	1.566	0.705	51.6	53.2	0.006	212	81
3.875	1.020	1.420	0.871	51.4	53.0	0.013	279	113
3.875	4.999	0.967	3.035	52.2	55.8	0.061	284	195
3.875	8.001	0.923	4.725	52.8	58.5	0.096	274	228
3.875	12.000	0.831	6.799	53.8	62.3	0.150	192	398
3.875	0.000	1.885	0.554	65.6	66.6	0.000	108	48
3.875	0.500	1.635	0.687	63.6	64.4	0.006	197	80
3.875	1.002	1.485	0.842	62.5	63.6	0.014	259	107
3.875	3.002	1.232	1.776	64.1	66.4	0.040	318	126
3.875	5.001	1.156	2.872	62.6	65.4	0.062	368	158
3.875	8.001	1.004	4.812	63.2	69.6	0.097	287	206
3.875	10.000	0.899	5.799	62.8	69.2	0.123	210	317
3.875	12.001	0.792	6.763	61.5	70.8	0.152	184	429

Table 13.12: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, eighth run)

Note: only V_{gs} was manually changed

The first two results were NOT used as $V_{gs} < V_{th}$, which led to incomparable f_{max} values

3.395	8.012	0.535	0.522	23.7	27.4	0.080	243	335
3.335	8.013	0.171	3.885	23.8	29.6	0.105	163	1315
3.875	8.014	0.534	3.756	23.6	30.1	0.100	148	383
3.674	8.014	0.199	3.952	23.8	31.2	0.104	171	1112
4.156	8.014	0.959	4.749	23.6	30.9	0.098	394	197
4.396	8.013	1.289	4.281	23.9	31.7	0.102	334	133
4.630	8.015	1.514	4.223	23.8	32.6	0.102	280	108
4.910	8.015	1.808	4.126	23.9	33.5	0.108	227	80
5.070	8.015	2.118	4.049	23.8	33.9	0.108	194	72

Table 13.13: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, ninth run)

Note: only V_{ps} was manually changed

4.670	3.001	1.821	1.640	23.2	25.0	0.040	245	72
4.670	2.009	1.994	1.200	23.0	24.6	0.028	204	62
4.670	1.003	2.300	0.840	23.1	23.5	0.014	142	54
4.670	0.504	2.440	0.712	23.4	23.6	0.008	100	42
4.670	4.008	1.567	2.160	23.5	27.0	0.054	250	81
4.670	5.009	1.497	2.720	23.3	28.2	0.067	270	90
4.670	6.003	1.427	3.240	23.0	30.0	0.081	290	93
4.670	6.991	1.380	3.760	23.2	31.0	0.094	302	98
4.670	8.004	1.285	4.440	23.5	33.0	0.108	317	105
4.670	0.304	2.498	0.664	23.1	23.9	0.006	97	47
4.670	0.252	2.560	0.648	23.4	24.7	0.004	93	34
4.670	0.099	2.600	0.616	23.3	24.8	0.002	95	31
4.670	0.048	2.637	0.608	23.0	24.5	0.002	95	29

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
4.670	0.000	2.715	0.584	23.0	24.6	0.000	94	23

Table 13.14: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, tenth run)

Note: only V_{ps} was manually changed, although T_a changed due to dissipation & the lamps

4.670	0.000	2.600	0.608	28.7	29.9	0.000	100	24
4.670	0.099	2.600	0.632	29.1	30.3	0.002	98	25
4.670	0.505	2.450	0.728	30.4	32.5	0.008	101	39
4.670	1.008	2.180	0.872	30.6	32.8	0.014	150	62
4.670	1.999	1.956	1.220	31.0	33.3	0.028	210	62
4.670	3.001	1.769	1.655	31.5	34.6	0.040	198	67
4.670	4.002	1.512	2.180	31.4	35.0	0.054	254	80
4.670	5.000	1.427	2.720	32.3	36.5	0.068	284	87
4.670	5.999	1.453	3.240	32.5	38.5	0.082	283	90
4.670	7.001	1.434	3.760	32.6	39.1	0.094	292	95
4.670	8.004	1.340	4.400	32.5	40.5	0.108	311	101

Table 13.15: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, 11th run)

Note: only V_{ps} was manually changed, although T_a changed due to dissipation & the lamps

4.670	0.000	2.735	0.592	40.9	43.1	0.000	98	24
4.670	0.053	2.693	0.600	41.2	43.7	0.001	97	26
4.670	0.099	2.600	0.616	41.8	44.3	0.002	95	32
4.670	0.302	2.560	0.656	42.1	44.8	0.004	95	36
4.670	0.505	2.450	0.704	42.8	45.4	0.008	97	44
4.670	1.000	2.210	0.840	42.8	45.5	0.015	143	69
4.670	2.004	2.010	1.200	43.3	46.1	0.028	197	57
4.670	3.001	1.819	1.640	44.2	47.2	0.042	190	68
4.670	4.002	1.625	2.154	43.8	47.9	0.056	233	76
4.670	5.006	1.581	2.661	43.8	48.5	0.068	252	82
4.670	6.003	1.606	3.163	44.2	50.0	0.082	251	85
4.670	6.996	1.457	3.732	44.1	50.8	0.096	284	91
4.670	7.998	1.410	4.326	44.0	52.6	1.090	295	94

Table 13.16: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, 12th run)

Note: only V_{ps} was manually changed, although T_a changed due to dissipation & the lamps

4.670	0.000	2.877	0.560	49.8	51.0	0.000	89	27
4.670	0.051	2.850	0.584	49.8	52.0	0.002	92	26
4.670	0.105	2.770	0.600	50.0	52.6	0.002	92	30
4.670	0.307	2.683	0.640	50.3	52.5	0.005	92	36
4.670	0.505	2.580	0.680	50.6	52.7	0.008	92	43
4.670	1.008	2.290	0.824	50.0	53.4	0.015	139	62
4.670	1.505	2.015	1.016	50.2	52.7	0.022	188	60
4.670	2.009	1.967	1.187	50.9	53.3	0.028	201	56
4.670	3.002	1.772	1.652	51.0	53.9	0.042	197	66
4.670	4.002	1.696	2.138	50.9	54.3	0.055	224	74
4.670	4.994	1.600	2.656	50.9	55.5	0.068	248	82

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
4.670	5.998	1.536	3.210	51.3	56.7	0.082	265	84
4.670	7.002	1.484	3.755	51.2	57.4	0.096	282	90
4.670	7.999	1.358	4.397	51.4	58.7	0.109	307	95

Table 13.17: f_{max} , V_{ds} , I_{ds} , T_c , t_{rise} & t_{fall} with varying V_{gs} , V_{ps} & T_a (third measurement, 13th run)

Note: only V_{ps} was manually changed, although T_a changed due to dissipation & the lamps

V_{gs} (V)	V_{ps} (V)	f_{max} (MHz)	V_{ds} (V)	T_a (°C)	T_c (°C)	I_{ds} (A)	t_{rise} (ns)	t_{fall} (ns)
4.670	0.000	2.996	0.536	62.8	65.1	0.000	81	21
4.670	0.052	2.864	0.568	60.8	62.3	0.002	90	26
4.670	0.104	2.840	0.576	59.8	61.5	0.002	92	28
4.670	0.302	2.740	0.616	59.6	61.2	0.006	92	34
4.670	0.503	2.638	0.672	59.1	61.0	0.008	91	40
4.670	0.997	2.334	0.816	58.9	60.8	0.014	135	62
4.670	1.504	2.226	0.984	58.8	61.0	0.022	171	57
4.670	2.009	2.046	1.187	58.9	61.2	0.028	203	58
4.670	3.007	1.957	1.617	58.5	61.2	0.042	204	64
4.670	4.003	1.822	2.102	58.8	61.2	0.055	210	72
4.670	5.001	1.707	2.621	58.3	62.2	0.069	232	79
4.670	6.003	1.609	3.158	58.6	63.2	0.082	253	85
4.670	7.001	1.589	3.695	58.7	64.6	0.094	259	87
4.670	8.005	1.507	4.258	58.6	66.2	0.108	275	91

Regression analysis

The settings contain, in order, the following variables: width of search (so for i in range($-n, n + 1$)), the resolution multiplier, the base of the resolution, the resolution exponent (start/end), the start values for (a/b/c/d/e/f), the individual dividers of the resolution for (a/b/c/d/e/f), the power of ($V_{gs} - b$).

The output contains, in order, the following values: the average of the square of the deviation, the optimal values for a/b/c/d/e/f).

Model 1

$n=1$ settings (12, 1, 10, (0, 8), (12, 3, 2.4, 25, 0.8), (1, 10, 5, 1, 2), 1)
 output (0.07567, (19.766, 3.483, 1.428, 37.333, -5.0166))

$n=2$ settings (12, 1, 10, (0, 8), (12, 3, 2.4, 25, 0.8), (1, 10, 5, 1, 2), 2)
 output (0.06563, (6.863, 2.831, 0.494, 38.033, -4.866))

Model 2

$n=1$ settings (12, 1, 10, (0, 8), (12, 0.1, 2.4, 25, 0.8), (1, 2, 5, 1, 2), 1)
 output (0.02928, (4.565, -0.476, -0.0326, 35.666, -4.866))

$n=2$ settings (12, 1, 10, (0, 8), (12, 0.1, 2.4, 25, 0.8), (1, 2, 5, 1, 2), 2)
 output (0.04816, (7.348, -1.493, 0.5357, 37.353, -5.366))

Model 3

$n=1$ settings (5, 2, 5, (0, 14), (12, 3, 2.4, 30, 0.9, 0.1), (1, 5, 5, 0.5, 2, 2), 1)
 output (0.03425, (5.500, 3.193, 0.1029, 22.679, -0.3500, -0.5368))

$n=2$ settings (5, 2, 5, (0, 14), (12, 3, 2.4, 30, 0.9, 0.1), (1, 5, 5, 0.5, 2, 2), 2)
 output (0.03543, (3.500, 2.724, 0.1873, 35.475, -4.350, -0.7500))

Table 13.18: the results of the regression analysis using the three models

The models used are equations (10.1), (10.2) and (10.3) for respectively model 1, 2 and 3.

The code (Figure 14.1) is configured using the settings of model 3 with $n=1$.

14. Code

This section contains the source code used to perform all regression analysis, followed by the code used to draw all 3d graphs. All code can be downloaded from <https://ttt.arnaudsaintgenez.kaspermuller.nl/>

Regression analysis

The following code was used to perform the regression analysis. While the code itself is not optimal, it is very flexible and versatile. `lowest_dev`, all the `range(a, b)` values, `deviation()` and the resolution multipliers can all be changed to customize the range, accuracy, function, etc.

```

from math import log

data = []                                #[Vgs1, Vds1, Tj1, Fmax1), (Vgs2, Vds2, Tj2, Fmax2), ...]

multiplied_by_zero = 0                   #Prevent 0 * .... = 0 from locking in the result
for datapoint in data:
    multiplied_by_zero += datapoint[3] ** 2
multiplied_by_zero /= len(data)

lowest_dev = (10**50, (12, 3, 2.4, 30, 0.9, 0.1))    #(big number, start_values(a, b, c, d, e, f))

def deviation(datapoint, variables):        #return (measured fmax - calculated fmax)^2
    R25 = (datapoint[0]-variables[1]) ** -1
    R = R25 * (datapoint[2]/300) ** variables[5] + variables[2]
    Fmax = variables[0] / (log(variables[3] * datapoint[1] + variables[4]) * R)
    return (datapoint[3] - Fmax) ** 2

for resolution_exponent in range(0,14):
    resolution = 2 * 5 ** (-resolution_exponent)    #Resolution ranges from 2 * 5^0 to 2 * 5^13
    starting_variables = lowest_dev[1]              #Start at the values previously given
    for i in range(-5,6):
        a = starting_variables[0] + i * resolution    #Try multiple values between old_a +- 5 * resolution
        for i in range(-5,6):
            b = starting_variables[1] + i * resolution / 5
            for i in range(-5,6):
                c = starting_variables[2] + i * resolution / 5
                for i in range(-5,6):
                    d = starting_variables[3] + i * resolution * 2
                    if d <= 0: d = 1
                    for i in range(-5,6):
                        e = starting_variables[4] + i * resolution / 2
                        for i in range(-5,6):
                            f = starting_variables[5] + i * resolution / 2
                            var_squared = 0
                            for datapoint in data:
                                var_squared += deviation(datapoint, (a,b,c,d,e,f))    #Calculate dev^2 for given variables
                            var_squared /= len(data)
                            if var_squared <= lowest_dev[0] and var_squared != multiplied_by_zero: #If new low, replace old values
                                lowest_dev = (var_squared, (a,b,c,d,e,f))    #Also check if not multiplied by 0

print('done', lowest_dev)                #Prints (var^2, (a, b, c, d, e, f))

```

Figure 14.1: the code used for the regression analysis

Drawing graphs

The following code was used to draw the main graphs. The code is written in M and should be runnable on both Matlab and GNU Octave. The code reads values and drawing options from an excel sheet.

```
# ===== Tweaking The Transistor ===== #
# Script to plot Fmax relations #
# By: Arnaud Saint-Genез & Kasper Müller #
# ===== Tweaking The Transistor ===== #

clear;
pkg load io;
fig = figure('name', 'Fmax');

# General Script Options
set(0, "defaultaxesfontname", "Georgia")
set(0, "defaultaxesfontsize", 12)
set(0, "defaulttextfontname", "Georgia")
set(0, "defaulttextfontsize", 12)

# Read data from sheet
[data] = xlsread('Fdata.xlsx');
Vgs = data(:,1);
Vds = data(:,2);
Ids = data(:,3);
Ta = data(:,4);
Tc = data(:,5);
Trise = data(:,6);
Tfall = data(:,7);
Fmax = data(:,8);

# Read regression constants from sheet
cA = data(1,10);
cB = data(2,10);
cC = data(3,10);
cD = data(4,10);
cE = data(5,10);
pT = data(6,10);
qT = data(7,10);
qV = data(8,10);
# Define fmax formula with constants
f = @(Vgs, Vds, Tj) cA ./ (((Tj ./ pT) .^ qT) ./ ((Vgs ./ cB) .^ qV)) .+ cC .* log(cD .* Vds .+ cE);

# Read Tj formula from other sheet
[Tdata] = xlsread('Tdata.xlsx');
tcA = Tdata(1,5);
# Tj formula
fTj = @(Ta, Tc) ((Tc ./ Ta) .* tcA) + Ta;
# Calculate Tj for each point.
Tj = fTj(Ta, Tc);

# Read regression graph options from sheet
precision = data(10, 10);
tjOverflow = data(11, 10);
VdsMin = data(12, 10);
VdsMax = data(13, 10);

# Read style options from sheet
markersize = data(15, 10);

# Read image options from sheet
image = data(17,10);
imageAzimuth = data(18, 10);
imageElevation = data(19, 10);
```

```

# Read & define chart bounds
plotVgs = data(:,12);
plotVgs = plotVgs(~isnan(plotVgs));
defT = linspace(min(Tj)-tjOverflow, max(Tj)+tjOverflow, precision);
defVds = linspace(VdsMin, VdsMax, precision);
[RegTj, RegVds] = meshgrid(defT, defVds);
# Color
caxis([min(Vgs), max(Vgs)]);

# Plot raw data
scatter3(Tj, Vds, Fmax, markersize, Vgs, 'filled', 'MarkerEdgeColor', [0,0,0]);
hold on;

# Legend reference.
legends = [];
hh = []; #Legend headers

# Plot regression graphs
for regVgs = plotVgs'
    legends = [legends; ['Vgs: ' num2str(regVgs, 3)] 'V'];
    # Calculate fmax for this Vgs:
    RegFmax = f(regVgs, RegVds, RegTj);
    # Color
    PlotColor = regVgs .* RegFmax ./ RegFmax;
    # Draw
    m = mesh(RegTj, RegVds, RegFmax, PlotColor, 'facealpha', 0.1, 'facecolor', 'none');
    #Fake draw for legend stuff.
    p = scatter(max(Tj), 0, markersize, regVgs, 'filled', 'MarkerEdgeColor', [0,0,0]);
    hh = [hh, p];
    hidden on;
endfor

# Hide fake draw with white scatter plot. Why not?
scatter(max(Tj), 0, markersize, [1,1,1], 'filled', 'MarkerEdgeColor', [1,1,1]);

# Set further chart options
legend(hh, legends);
title('Relations to Fmax');
xlabel('Tj (K)');
ylabel('Vds (V)');
zlabel('Fmax (MHz)');

view([imageAzimuth, imageElevation]);

# Finish to save.
hold off;

# Save graph image if specified
if (image > 0)
    saveas(fig, ['snapshot-' num2str(image)], 'png')
    #print(['snapshot-' num2str(image)], '-dsvg');
endif

```

Figure 14.2: the code used to draw the graphs.

15. Research Journal

- *2018-04/05* : Arnaud & Kasper: Started with theorizing about the research. Spent some time during train journeys talking about computers and performance.
- *2018-05-11* : Kasper & Arnaud: 2 hours : First research proposal is put on paper. Original idea is to look at the different ways computer performance can be achieved.
- *2018-05-24* : Arnaud : 2 hours : Looked up multiple sources about computer performance and transistors.
- *2018-05-29* : Arnaud & Kasper : 0.2 hours : Discussed paper layout.
- *2018-05-31* : Arnaud & Kasper : 1.5 hours : Arnaud formed sections based on the discussed ideas. Kasper did some research about Moore. Consensus is reached about the topic and ideas. Main questions are formed about optimal performance. For instance: What is the physical limit of transistor size? What is the economic (optimal) limit of transistor size? What is the optimal ratio between the size of chipsets and the amount. There is an idea to look at case studies such as the intel lake processor to exemplify microarchitectures and the Intel 10 nm to exemplify transistor size. The idea about a 'main' experiment is to construct a cluster computer and look at performance of chipsets. Though most of the paper would be literature study.
- *2018-05-31* : Arnaud & Kasper : 1.5 hours : After having discussed with our research mentor, we came to the conclusion that our research proposal was a bit wide. Quantum computers are scrapped almost immediately, but the other subjects soon followed leaving us eventually with the question about the physical limits of transistors.
- *2018-06-06* : Arnaud & Kasper : 1 hour : Arnaud proposes new subquestions and formulates small hypothesis. Kasper reviewed and changed some sections.
- ⇒ *2018-06-10* : The research proposal is send to our research mentor (Mirjam Marseille)
- *2018-06-25* : Arnaud : 4 hours : Started work on the paper. Theory about semiconductors is written as well as the initial research method of two experiments; the effect of voltage on the maximum frequency and the effect of temperature on the frequency. A hypothesis for the first experiment is written.
- *2018-06-27* : Arnaud : 4 hours : Researched and wrote theory about the transistor and how quickly it fills up.
- *2018-06-30* : Kasper : 8 hours : Researched Mosfet and rehashed electronics. Reviewed Arnaud's content and worked out initial theory about diodes, transistors and mosfets.
- *2018-07-01* : Kasper : 2 hours : Moved around some content, researched capacitors and impedance. Wrote theory about capacitors.

- *2018-07-01* : Kasper & Arnaud : 3 hours : Kasper finished theory on impedance in capacitors and wrote hypotheses on temperature. Arnaud wrote research method and hypothesis on impedance in mosfets. For now all hypothesis are subject to change. This must be seen as the initial worked-out thought we have.
- ⇒ *2018-07-01* : Theory , hypothesis and research method is sent to Mirjam our research mentor for first examination.
- *2018-07-02* : Kasper : 1 hour : Started work on presentation for first pitch at school. Wrote the research journal up to now. From now on this will be kept up to date as work is being done.
- *2018-07-03* : Kasper : 1 hour : More work on the school presentation. Researched oscilloscopes.
- *2018-07-04* : Kasper : 2 hours : Finished first presentation and gave it to the two Research mentors (Wim Hendrikson & Carin Heere) at school.
- ⇐ *2018-07-05* : Mirjam Marseille communicates feedback about the work up till now.
- *2018-07-07* : Kasper & Arnaud : 0.5 hours : Read feedback and wrote a response / question. Mirjam Marseille seems content with the research but is seeing a strong lack in connections. Why are we addressing all those things in theory? What are the relationships between them? For now a FAIL is given and we need to address the issue. In our eyes we mainly need to patch up a few references, move some theory around, and stress the importance of the theory in an introduction section. This would also include the reasons for doing this research (which we have already thought about). Just as important for now is to come up with a main research question that encompasses the experiments we want to perform. All these things are the next step and will be addressed in the upcoming weeks.
- ⇐ *2018-07-26* : Mirjam Marseille communicates feedback on proposal for changes. She communicates that the research should follow out of the question instead of vice versa.
- *2018-08-25* : Kasper : 0.5 hours : Puts review comments into drive document. Fixed some sentences. In consultation with Arnaud the research question is reformulated to now be: "To what extent can voltage, temperature and impedance be used to predict peak switching frequency in mosfet transistors."
- *2018-08-26* : Arnaud : 1.2 hours : Reworked much of hypotheses / methodology to fit the new research question. Hunted for errors in written hypothesis.
- *2018-08-27* : Arnaud & Kasper : 1 hours : Arnaud did more work on reworking methodology and sub-questions. Kasper wrote part of the introduction.
- *2018-08-28* : Kasper & Arnaud : 3 hours : Kasper fixed minor mistakes in theory, addressed all comments in theory and wrote more of the introduction. Arnaud researched the effects of impedance.
- *2018-08-29* : Arnaud & Kasper : 5 hours : Both reorganised theory. Arnaud shortened introduction. Kasper shortened hypothesis. Further patches were made.
- ⇒ *2018-08-29* : Patched research proposal is sent to Mirjam.
- ⇐ *2018-08-30* : Mirjam communicates that the points have been addressed to such an extent that a pass is granted.
- *2018-09-09* : Kasper : 2 hours : Fixed most of the points from Mirjam's feedback. General theory introduction still needs to be written.
- ⇒ *2018-09-09* : Patched research proposal is sent to Mirjam.
- ⇐ *2018-09-13* : Mirjam gave feedback in person. She expressed that she was impressed by the amount of improvements that have been booked in the retake and in the last week. Mirjam says she thinks it would be good if an overhaul occurs: remove all theory and start with a blank slate. Then copy over only the necessary points. For the next deadline the theory has

to be tightened and every step in the experiment should be described in detail so the only thing left to do is to conduct the actual experiment and process the results.

- *2018-10-01* : Arnaud & Kasper : 4.5 hours : Drastically reduced the theory to support the hypothesis and not much more. The old theory is cloned to an appendix document for reference purposes and to allow a detailed read on transistors. Detailed instructions on methods have been written and a general method setup is drafted. Name is changed to Tweaking the Transistor for a more accurate description of the research at hand.
- ⇒ *2018-10-01* : Research proposal is send to Mirjam. This is the last version of the proposal. We will now receive a GO / NO GO for performing the actual research.
- *2018-10-03* : Arnaud & Kasper : 2.5 hours : Created pitch for performing the research.
- ⇒ *2018-10-04* : Presentation about the research proposal is given.
- ⇐ *2018-10-04* : A No-Go is given to the proposal. Feedback is returned. Mirjam expresses that theory is a lot better in its current form (although a bit of the wording has to be improved). The reason for the no-go is a lacklustre methodology. Not enough details about practical feasibility are presented. For instance: inside (die) temperature will be hard to measure and we cannot just use an ordinary oscilloscope. We will get 2 weeks to present our next plan.
- *2018-10-09* : Arnaud : 3 hours : Researched measurement methods for junction temperature and equipment needed.
- *2018-10-10* : Arnaud & Kasper : 0.75 hours : Constructed email to ask for equipment from university departments.
- ⇒ *2018-10-11* : Send email to department of nano electronics. Called department of nano electronics to ask the same question.
- ⇐ *2018-10-12* : Received word from department of nano electronics, they have a capable oscilloscope, but no function generator.
- ⇔ *2018-10-12* : Kasper asked at his work and they happen to have ~~most~~ all instruments we need.
- *2018-10-19* : Arnaud & Kasper : 4 hours : Fixed many sentences in theory. Created new diagram for methodology & expanded on details.
- ⇒ *2018-10-19* : Improved theory and methodology are sent to Mirjam.
- ⇐ *2018-10-23* : Mirjam communicates feedback in person. A GO is given for the methodology, although she is still very sceptical about the way we want to determine on-die temperature. She likes us to perform a pilot within about a week.
- *2018-10-26* : Kasper : 3.5 hours : Constructed “ghetto” measuring circuit and hooked up circuitry. The plan was to measure peak performance of a IPP057N06N3 G Power Mosfet. Measuring went ok for ranges up to a few about a MHz, but round 1/7 of the peak frequency as per the data sheet (~1MHz) the signal started to be distorted to such an extent that it is indeterminable to tell if the transistor has already broken down. The same experiment will be conducted next friday but with a more carefully constructed circuit.
- *2018-11-02* : Kasper & Arnaud : 6 hours : constructed a circuit that was a lot less “ghetto”. Shorter ground leads lead to a more accurate measurement, allowing us to track signals up to 1.7 MHz. Still much rinkle / distortion is seen, prompting us to design a circuit board to further improve the circumstances of the measurement.
- *2018-11-09* : Kasper : 6 hours : Learned to work with kicad.
- *2018-11-11* : Kasper : 4 hours : Kasper designed PCB for measuring so we can lower the interference even more.
- *2018-11-13* : Kasper & Arnaud : 4 hours : Improved methodology, readied the document to be shared with peer review group.
- ⇒ *2018-11-13* : Document is send to peer review group.

- *2018-11-15* : Kasper : 3 hours : Learned to work with kicad.
- ⇔ *2018-11-15* : We received feedback from the peer review. Both Mirjam and Sander talked with us afterwards about the feasibility of impedance. We have decided that it might better to cross 'the odd one out' and focus purely on environmental factors. About environmental factors, we convinced Mirjam and Sander to go ahead and try to extrapolate the junction temperature using the method Arnaud described.
- ⇔ *2018-11-23* : Received PCB ordered from JLCPCB circuits!
- ⇔ *2018-11-26* : Received components ordered from Farnell.
- *2018-11-27* : Kasper : 1.5 hours : assembled pcb.
- *2018-11-30* : Arnaud & Kasper : 9 hours : Tested pcbs: interference is now negligible! Much practical work and learning was done. René gave us a thermal camera once he saw what we were coming up with. We performed many dry-runs on the pcbs and got ourselves familiar with another different oscilloscope. We also performed a temperature pilot using the resistance method. This went well, but ultimately gave us bad result, but also an insight how to perform the test properly next time.
- *2018-12-01* : Arnaud & Kasper : 3.5 hours : Worked on methodology. Calculated with results of temperature pilot study and the like. Arnaud wrote a part about the temperature results and Kasper wrote more about the PCB design process. Some parts of the theory were fixed and a new layout for the methodology was inserted to be more in line with the rest of the document.
- ⇒ *2018-12-01* : Concept document is sent to Mirjam.
- *2018-12-05* : Kasper : 2 hours : Worked on the presentation for the second pitch.
- *2018-12-06* : Arnaud : 1 hours : Finalised presentation, put data into graphs.
- ⇒ *2018-12-06* : Pitch is given containing only the pilot results and explanation of how future hurdles would be overcome.
- ⇔ *2018-12-06* : We received feedback via mail. Again a no-go was given. Bitterness is expressed because we began with the hardest variable, temperature (against apparent recommendation), a heat gun was used instead of an oven and the part about impedance had not been taken out the paper yet. We thus 'ignored feedback' The bad feedback was mostly because of our attitude, which was said to be too self-assured, stubborn and uncritical. The real experiment hasn't been done yet and is in their eyes excused away as only a 'formsache'.
- ⇒ *2018-12-09* : A response is drafted and sent to voice our take on the failure of last week. We did not expect the feedback and disagreed on a few points. We began with the hard variable, because we thought it would be the hardest to overcome, thus requiring the most time. Also being able to measure junction temperature was paramount to being able to make regression work later on. In methodology we had already said we were planning to use a fridge/oven and this was to be done next experiment day. We hadn't come around to removing everything relating to impedance yet since some stuff in there might be needed elsewhere. We tried expressing that the road ahead was still long and humbly (but that it would perhaps be easier than the hurdles we overcame so far).
- ⇔ *2018-12-14* : Mirjam send feedback to our feedback to her feedback. She was happy we responded. She reiterated some of the feedback and we now have the feeling we can move on.
- *2018-12-14* : Kasper & Arnaud : 4.5 hours : The oven-fridge was prepared and the complete setup was tested. The temperature measurements were redone as a test, this time making use of the oven-fridge. More sensible results were achieved this way. At the end however one of the transistors legs broke off. The soldered ghetto connections combined with

bending of the alligator clips proved to be too much. Since there was no way of reconnecting the transistor the measurements were useless.

- 2018-12-21 : Kasper & Arnaud : 6 hours : The temperature measurements were redone. This time a more accurate voltmeter was used. Measurements seemed to be consistent and were really usable for the first time. At the end of the second experiment however, we were rearranging a circuit when probably some capacitors in the power supply charged, because upon reconnecting it took some time for the current-limiter to kick in and transistor gave a large audible and visible spark. Upon performing basic tests it smoked out and off resistance had dropped into the range of a few ohms. Since these measurements are very transistor-dependent they can also be discarded for the main experiment. The data might however still be useful to compare temperature properties between these two transistors.
- 2018-12-27 : Arnaud & Kasper : 9 hours : Temperature measurements were redone completely using a new transistor. Basic max frequencies were tested at varying source-drain voltages. The setup was prepared for the next time. Arnaud stayed at Kasper's for some more bonding time.
- 2018-12-28 : Kasper & Arnaud : 9 hours : The real main experiment was conducted for the first time. Inside the designed chip many measurements at varying V_{gs} and V_{ds} were taken. Results produced graphs that might just be the relation we expected. After having taken many measurements we wanted to start varying temperature as well. The cables we wanted to use, however, were too short to reach the fridge. When connecting longer cables we might expect a bit lower maximum-frequency because of an increase in capacitance in the cable or something, but still at room-temperature we suddenly measured an increase of about 2.5 times the maximum frequency. This result completely unsettled us. Upon checking we concluded the only different factor was the cable. We checked all cables and it happened we took the only unmarked coax-cables around. All 50Ω impedant cables showed the same sudden increase in F_{max} ... The cables we used before were clearly not right for the job. For validity, we will have to redo everything from this day.
- 2018-12-31 : Kasper & Arnaud : 4 hours : Redid the main experiment. Varied with both V_{gs} , V_{sd} and T_a .
- 2018-12-31 : Arnaud : 4 hours: While Kasper had another appointment, Arnaud continued collecting data points.
- 2019-01-02 : Kasper : 8 hours : Collected more data points.
- 2019-01-06 : Arnaud : 5 hours : Transformed docs document to word, started rewriting theory and methodology.
- 2019-01-06 : Kasper & Arnaud : 7 hours : While Kasper created new circuit diagrams and tables of the results and research 3d graphing, Arnaud continued with writing methodology, theory and results. Arnaud also worked on the regression program.
- ⇒ 2019-01-07 : A very rough sketch of the paper is sent. In current form quite in disarray, but the main jist of the new paper index should be visible.
- 2019-01-07 : Arnaud : 3 hours : Continued on rewrite of paper.
- 2019-01-08 : Arnaud & Kasper : 4 hours : Arnaud continued with layout and rewrite of the theory. Kasper worked on restructuring methodology.
- 2019-01-10 : Arnaud & Kasper : 3 hours : Arnaud worked on data analysis and theory. Kasper finished rewriting sections about equipment and measurements.
- ⇐ 2019-01-10 : We received feedback from Mirjam in person. There was one concern about the subject matter: we took the temperature / resistance relation as a linear one, this was not in line with the spec sheet and with the theory. Further research needed to be done in

this area. She expressed that she thinks we can 'pull it off'. We also discussed our attitude towards this whole enterprise. We think we now understand each other's positions.

- 2019-01-11 : Arnaud : 2 hours : Researched better theoretic foundations temperature / resistance relations.
 - 2019-01-12 : Kasper : 2 hours : Worked on fixing and updating the research journal. (Re)searched papers regarding temperature / resistance in semiconductors and mosfets.
 - 2019-01-12 : Arnaud : 4 hours : Worked on layout and source indication. Also did research on temperature/resistance and compiled theory explaining the found relation. Researched how Vds influences Ids.
 - 2019-01-13 : Kasper : 1 hour : Update research journal.
 - 2019-01-14 : Kasper & Arnaud : It is decided to take a break and focus on school tests first.
 - 2019-01-18 : Kasper : 4 hours : Research working of octave / matlab, drew initial 3d graphing of data & regression formula. Drawing the regression reveals a sub-optimal fit, probably we are still missing an offset in the Vds.
 - 2019-01-19 : Kasper & Arnaud : 2 hours : Discussed about the offset, did more statistics and changed regression.
 - 2019-01-20 : Arnaud & Kasper : 4 hours : Discussed and researched temperature relations that still raise questions. For instance, what exponent to use to fit the temperature / resistance graph? Why does the temperature delta divided by the power output in the transistor not stay the same? In the end we have decided to use an exponential fit on the temperature graph that matches best what is described in established theory. Our formula differs in offset, which is explainable by the way the resistance was measured.
 - 2019-01-20 : Arnaud : 2 hours : improved regression program and performed more regressions to find better fits.
 - 2019-01-21 : Kasper & Arnaud : 5 hours : While Arnaud fixed the regression and worked on implementing the decided temperature formulas, Kasper worked on a sketch for the acknowledgements and abstract.
 - 2019-01-21 : Arnaud : 4 hours : Fixed excel sheet, made properly formatted tables for use in experiment.
 - 2019-01-22 : Arnaud & Kasper : 6 hours : Arnaud fixed some incorrect values still present in the regression. Kasper worked on getting Octave to draw proper diagrams.
 - 2019-01-22 : Arnaud : 7 hours : Worked on paper, fixed many layout issues, wrote about regression. Did more general data analysis.
 - 2019-01-23 : Kasper : 3 hours : Drew final graphs in Octave. Worked on research journal.
 - 2019-01-23 : Kasper & Arnaud : 6 hours : Did everything that needed to be done. Rewrote many parts of the method, results, discussion. Dotted some of the the i's and crossed the t's.
- ⇒ 2019-01-23 : The 'final' university version of the paper for is sent to Mirjam Marseille.
- 2019-01-25 : Kasper : 4 hours : fixed many small mistakes and marked many layout errors. Created a website to host regression and graphing code. This website will also function as hub for this paper.
 - 2019-01-25 : Arnaud & Kasper : 2 hours : Kasper worked more on patches throughout the document while Arnaud worked on improving the discussion.
 - 2019-01-26 : Arnaud : 3 hours : Fixed and added many parts to discussion.

- *2019-01-26* : Kasper & Arnaud : 7 hours : Bug fixes. Wrote reflection, abbreviation table, better abstract. General improvements.
- ⇒ *2019-01-26* : The final version is sent to Mirjam, Wim and Sandra. The paper version of this research is done.